Modeling the Emergence of Patterns

L-Systems and Beyond



Provost's Colloquium Series
East Stroudsburg University
September 20, 2023
Time and Location: 5.30pm to 7.00pm, Stroud Hall 117

Presented by:
Dr. Jey Chelladurai
College of Arts & Sciences
Computer Science Department
Assistant Professor
East Stroudsburg University of Pennsylvania

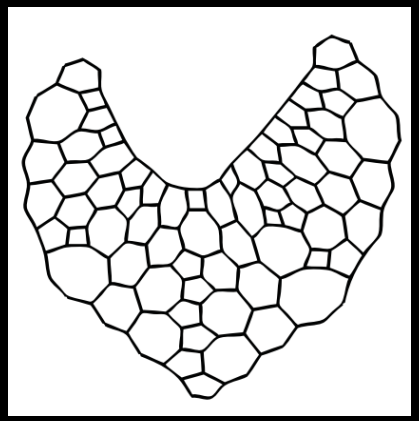
About Myself

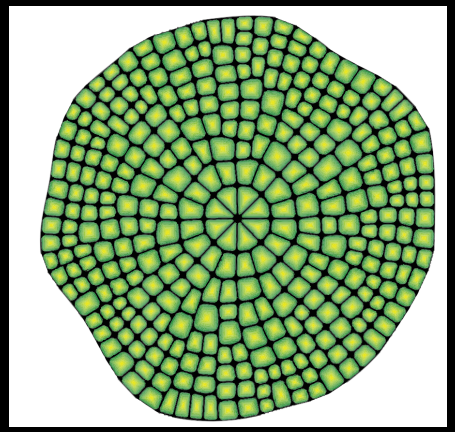
B.Tech – University of Madras (Information Technology)

MSc – University of Northern British Columbia (CPU scheduling algorithms). Supervised by Dr. Alex Aravind

PhD – University of Calgary (Used graphics and animation techniques to understand emergent forms in growing plant cellular tissues). Supervised by Dr. Prusinkiewicz, a pioneer in Computer Graphics/Tree modeling, 1997 SIGGRAPH AWARD RECEIPINT. Visit: www.algorithmicbotany.org

Emergent Patterns











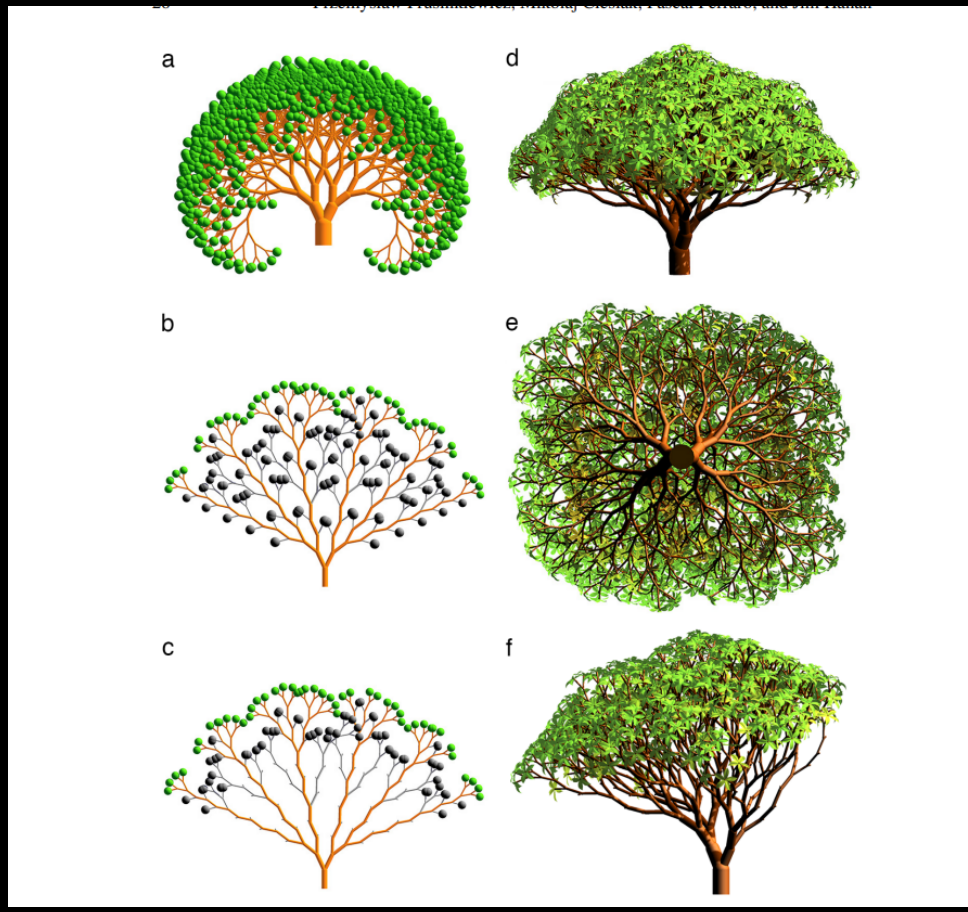


Image Courtsey: www.algorithmicbotany.org

• Intro to L-systems



Aristid Lindenmayer:

- Mathematical models of cellular interaction in development
- Journal of Theoretical Biology (1968)

Question:

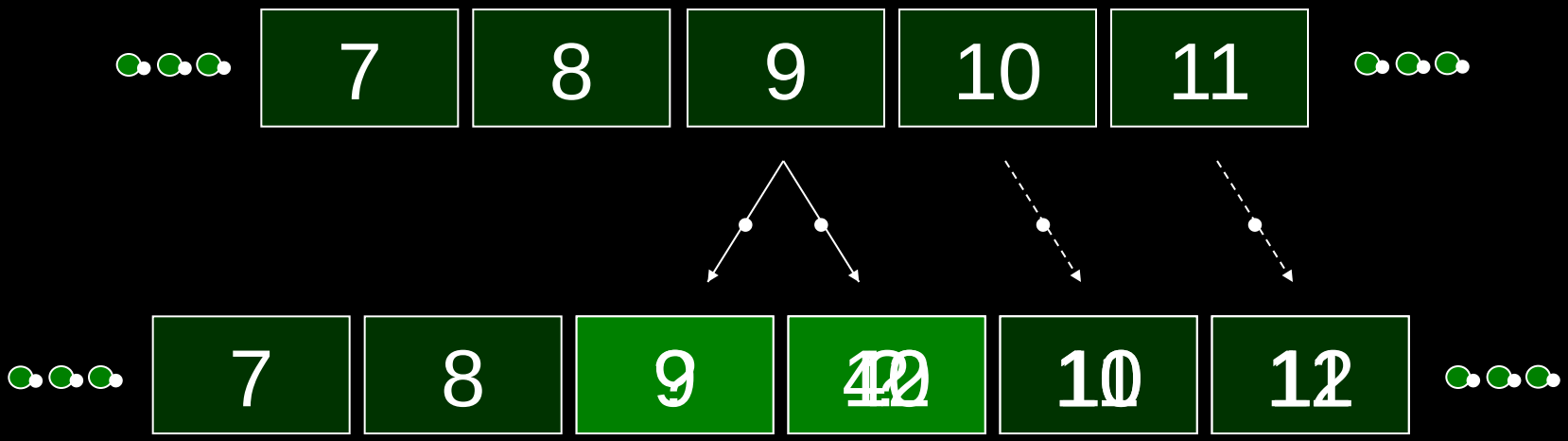
- Why was a new formalism needed?

- New formalism required



- “A developing organism is a dynamical system with a dynamic structure” (Giavitto and Michel, 2001)
-
- Existing formalisms not well-suited to handle this dynamic structure

- Problems with indices



Managing indices quickly becomes a headache

• Formal definition

A DOL-system is a triplet:

$$G = (V, \omega, P)$$

where:

V is a set of symbols, called the alphabet

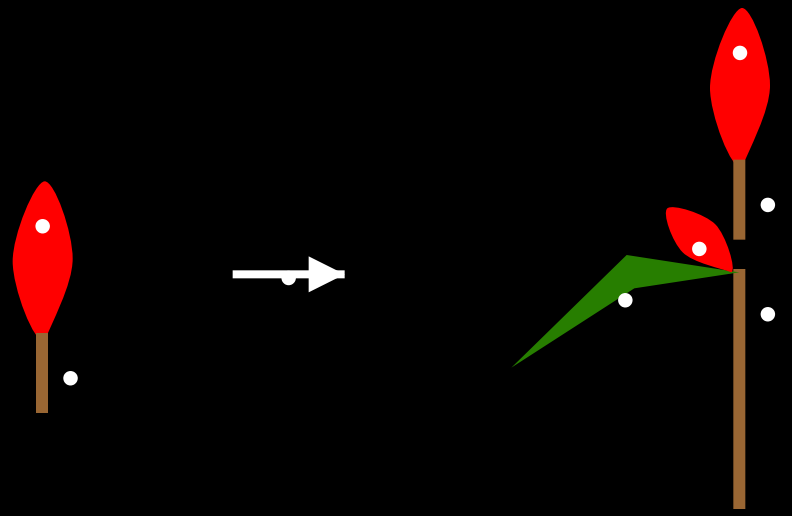
$\omega \in V^*$ is a string over V, called the axiom

$P \subseteq V \times V^*$ is a finite set of productions

Instead of writing (a, χ), we write $(a, \chi) \in P$ $a \rightarrow \chi$

Productions are applied in parallel!

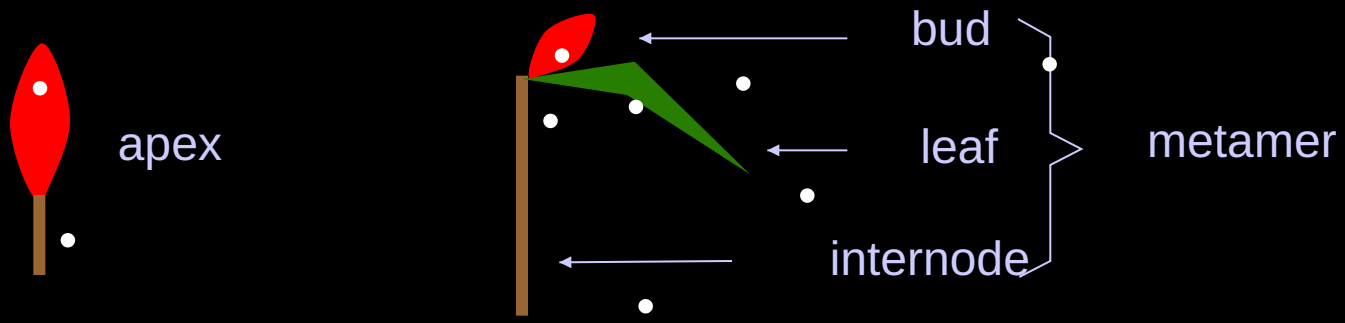
• Development as a rewriting process



production

- Development can be thought of as a re-writing process
- Apex is replaced by an internode, leaf, bud, and another apex

- Development as a rewriting process



L-System

Alphabet: (A, M)

Axiom: A

Production: A → MA

- Development as a rewriting process

L-System

Alphabet: (A, M)

Axiom: A

Production: A → MA



A

- Development as a rewriting process

L-System

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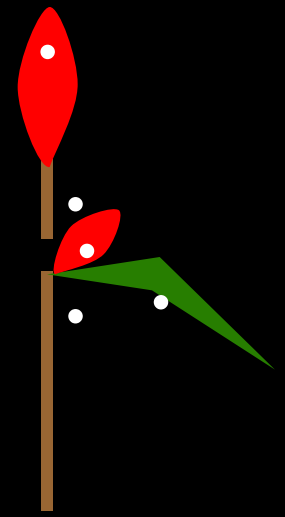


MA

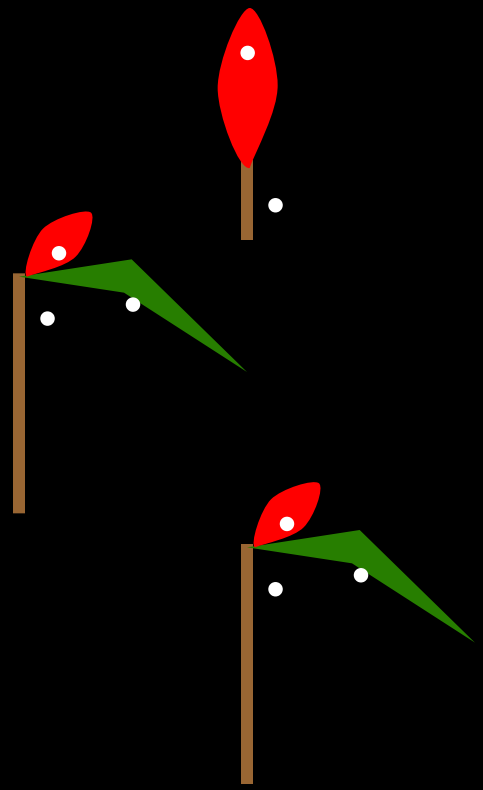
- Development as a rewriting process

L-System

Alphabet: (A, M)

Axiom: A

Production: A → MA



MMA

- Development as a rewriting process

L-System

Alphabet: (A, M)

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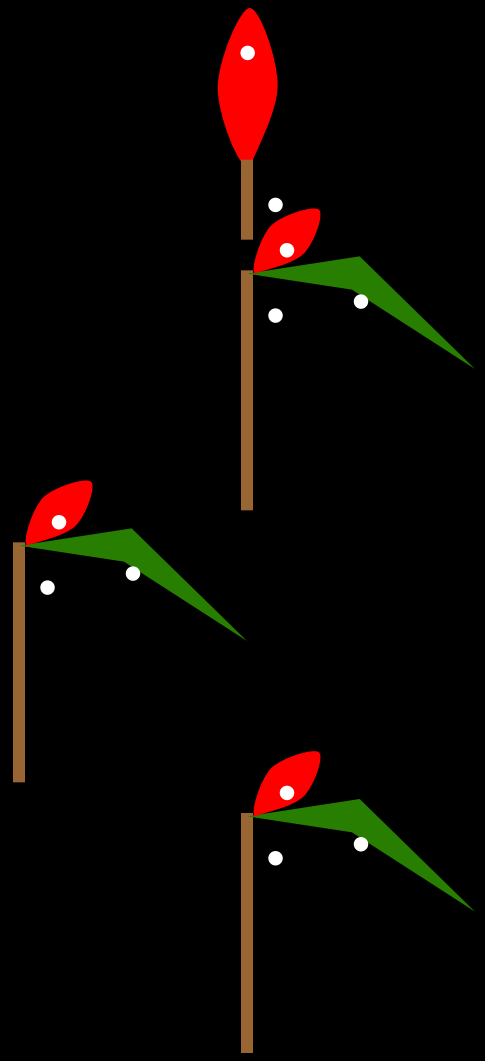
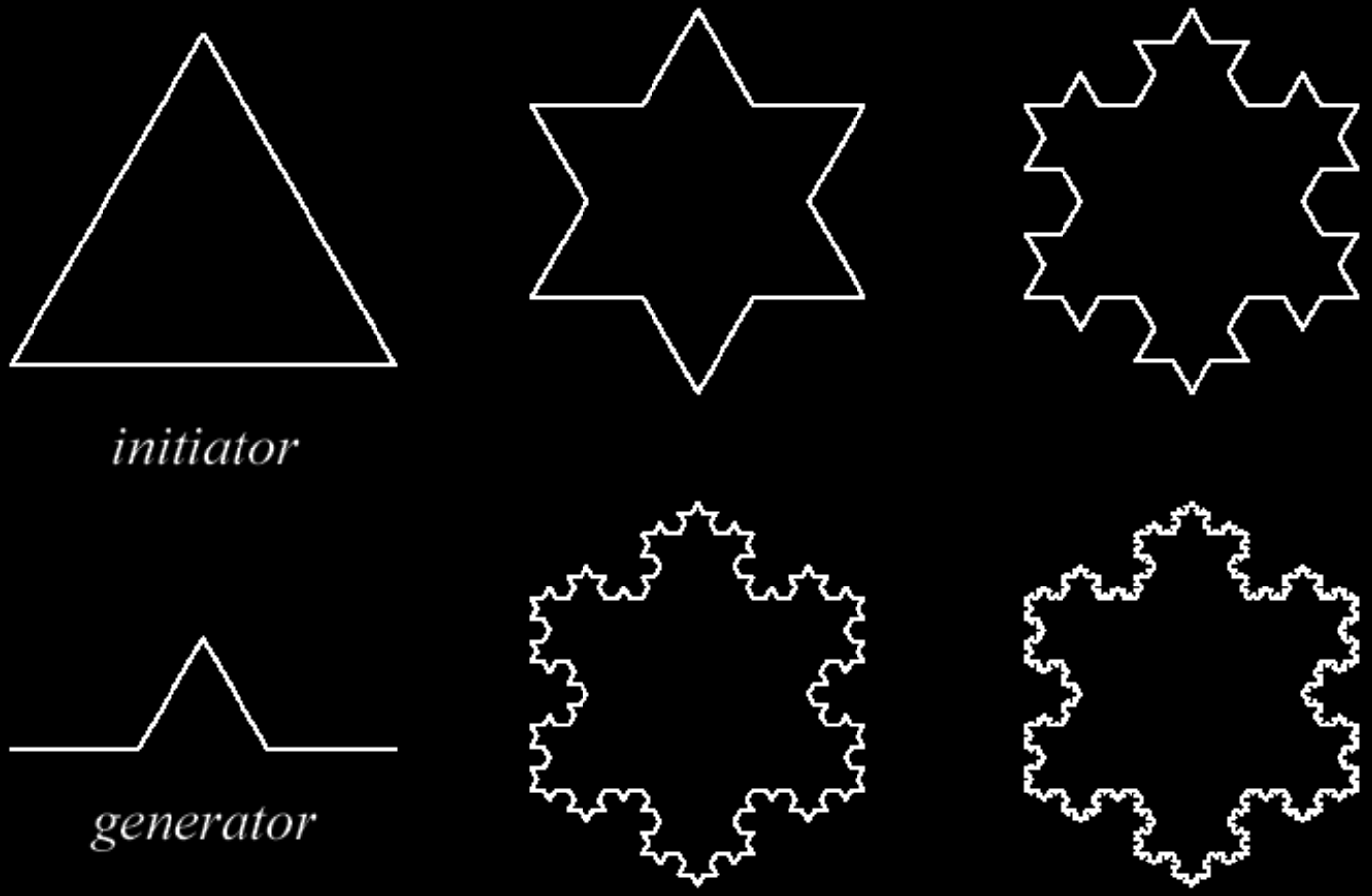
Production: A → MA



MMMA

- L-Systems for simple fractals

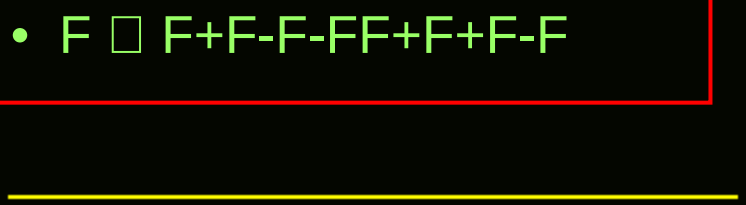


Snowflake Curve - Proposed in 1905 by von Koch

- L-systems are re-writing systems, based on formal language theory
- Define complex objects by successively replacing parts of a simpler object

Turtle interpretation of L-systems

- -F
- F □ F+F-F-FF+F+F-F



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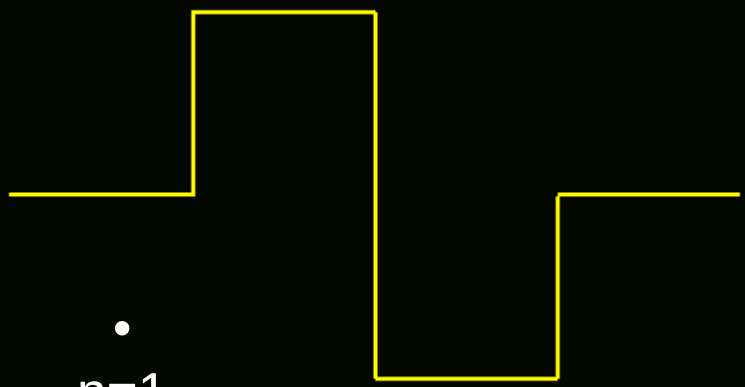

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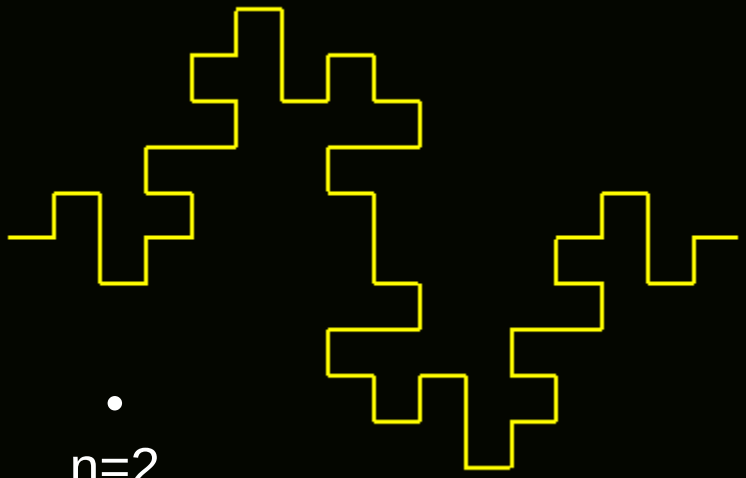

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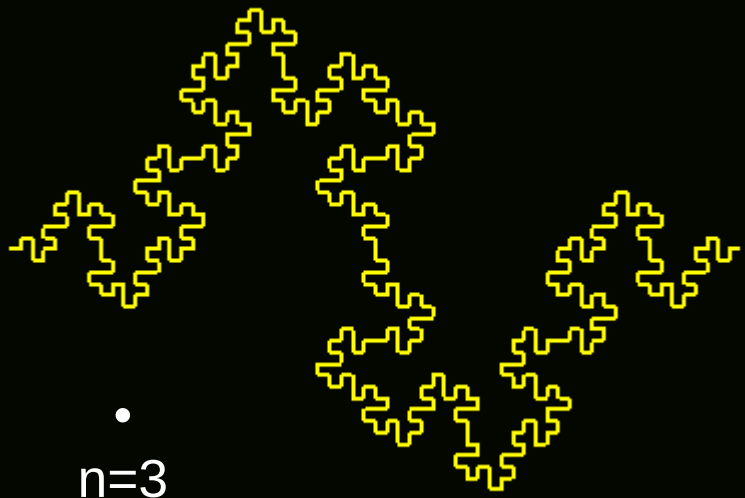
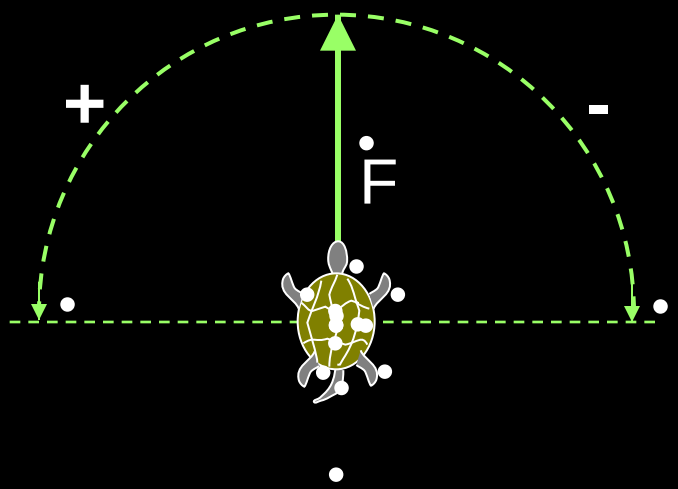


n=3

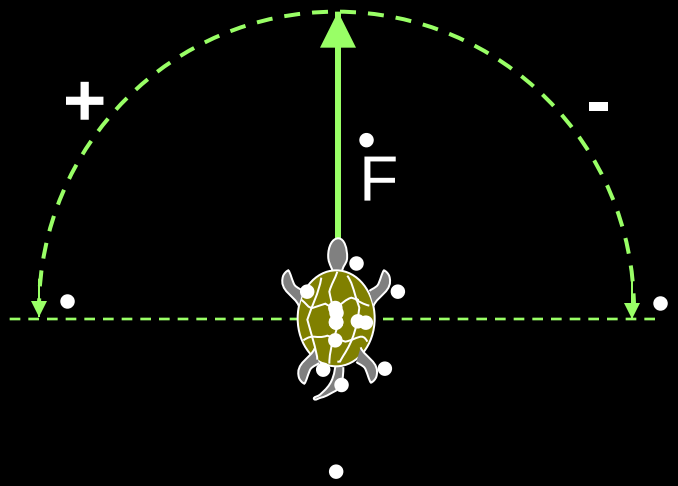
• Turtle interpretation of strings



Logo style turtle interpretation

F Move forward, drawing a line
+ Turn left 90 degrees
- Turn right 90 degrees

Turtle interpretation of strings



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F-F+F-FF-FF-F-F

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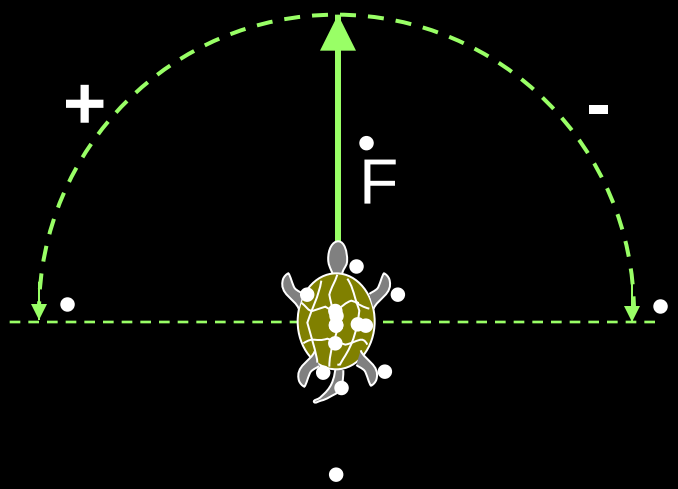



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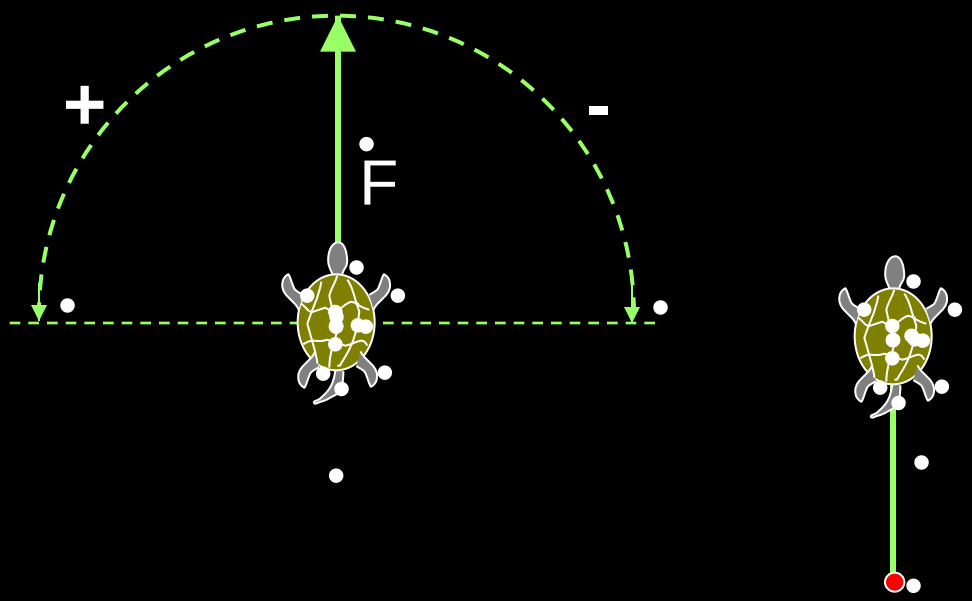

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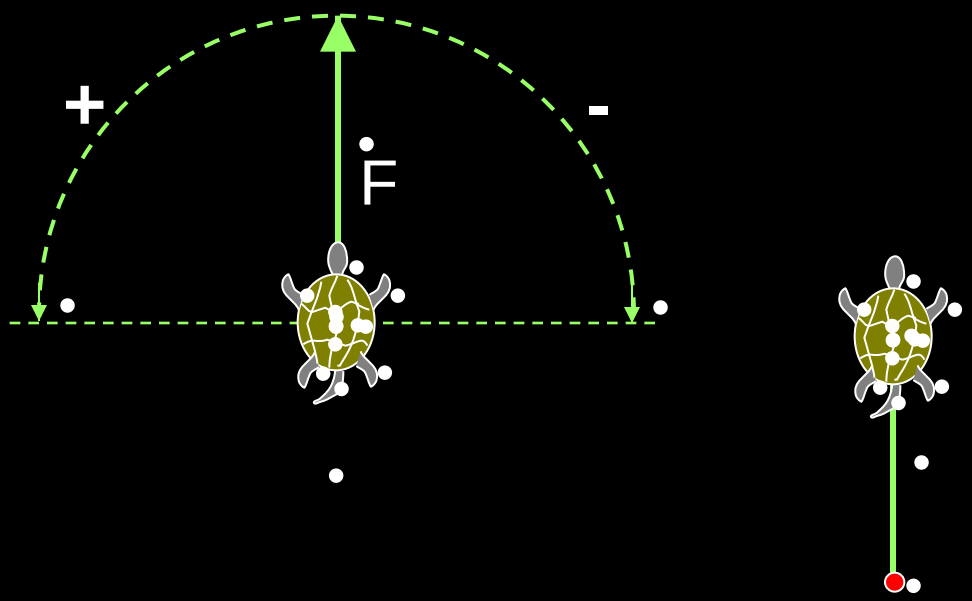

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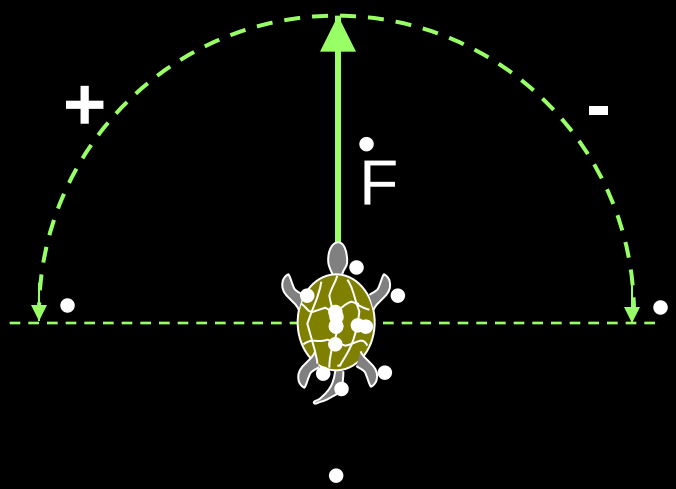

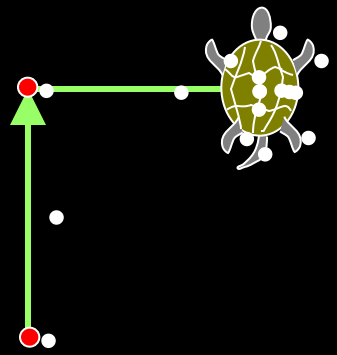

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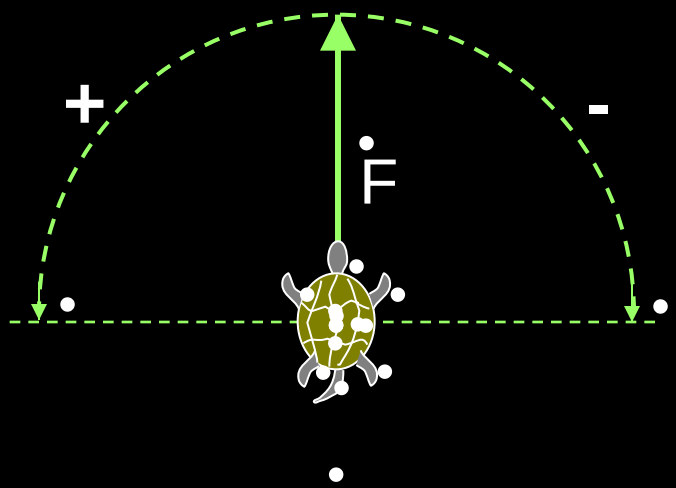

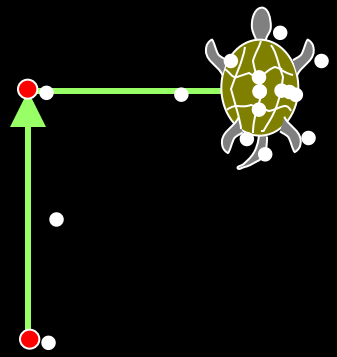

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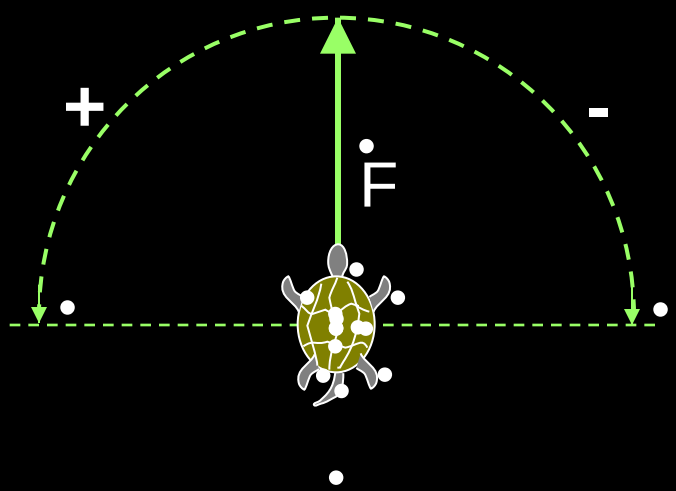

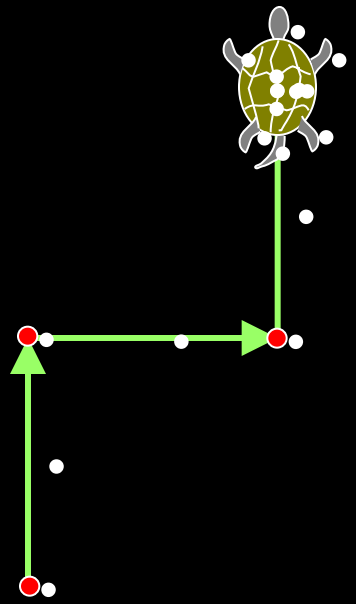

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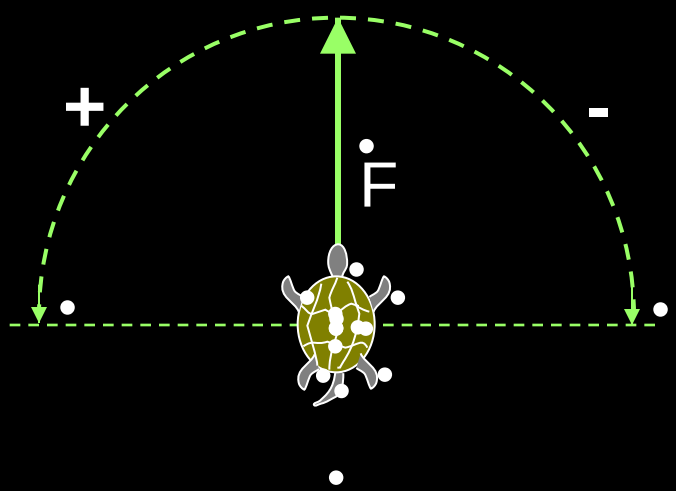

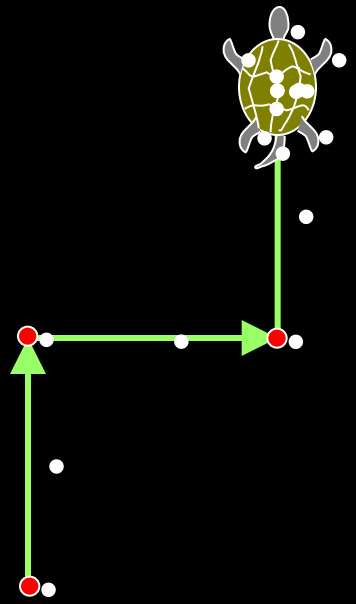

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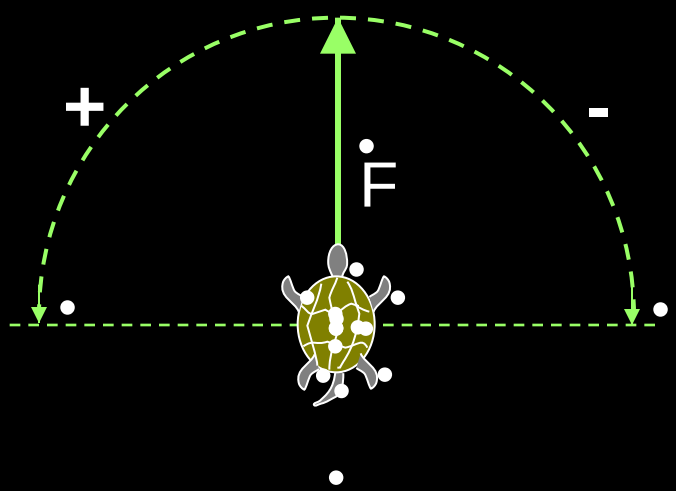

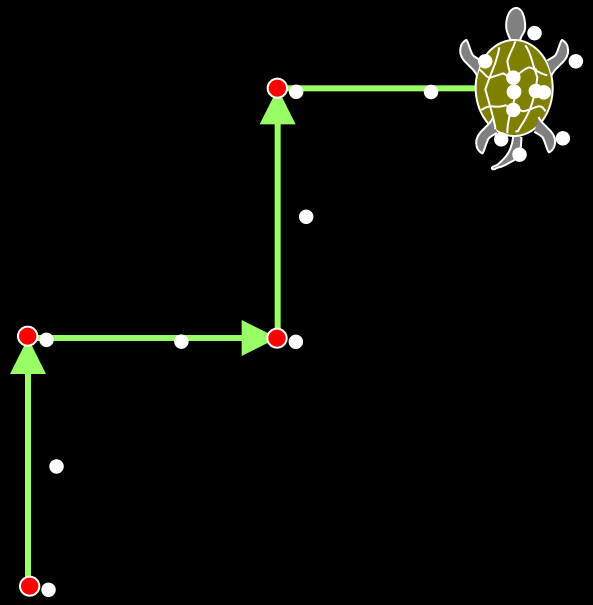

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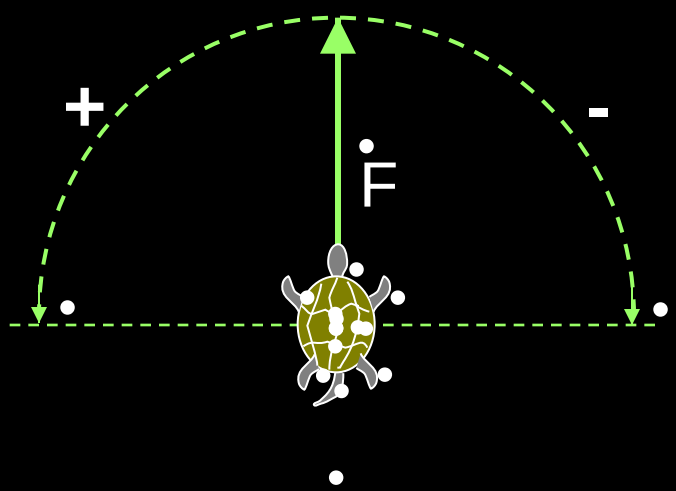

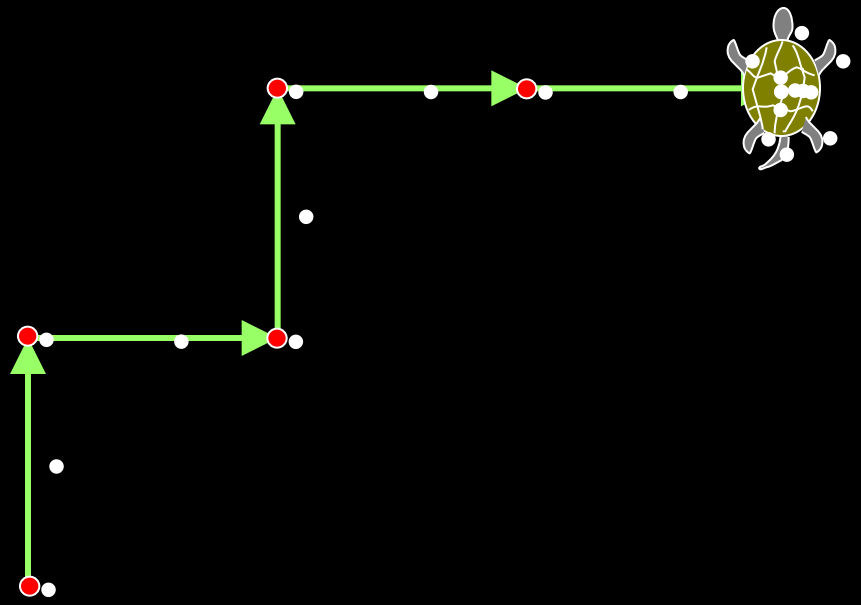

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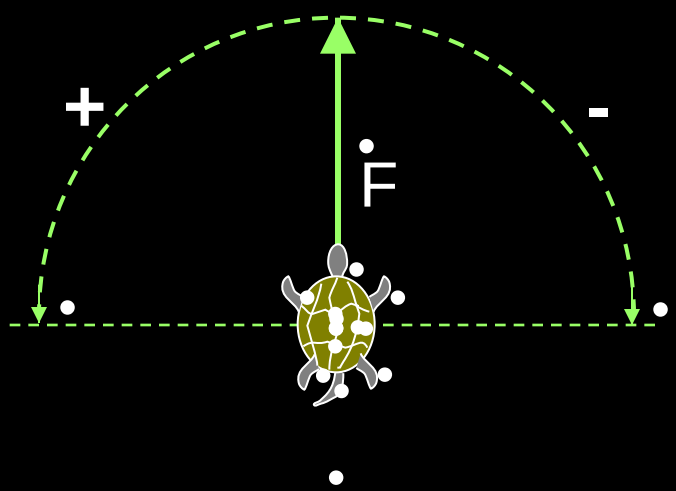

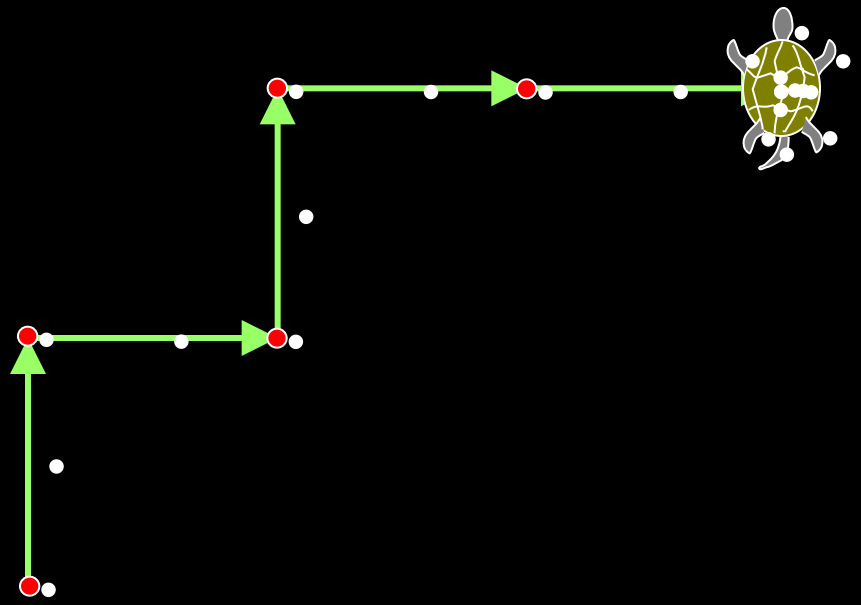

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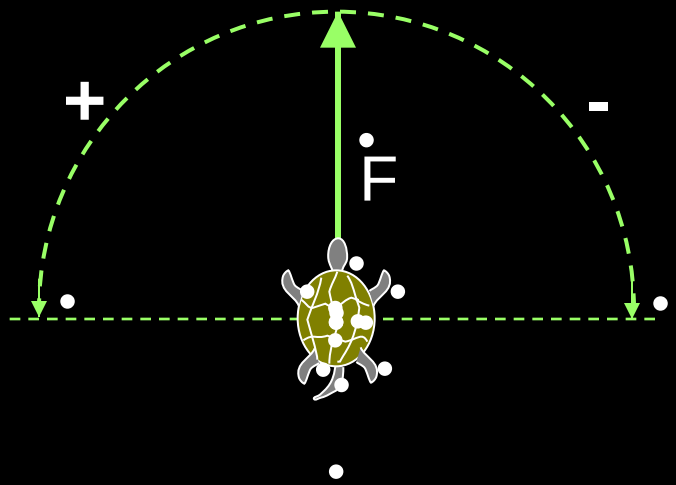

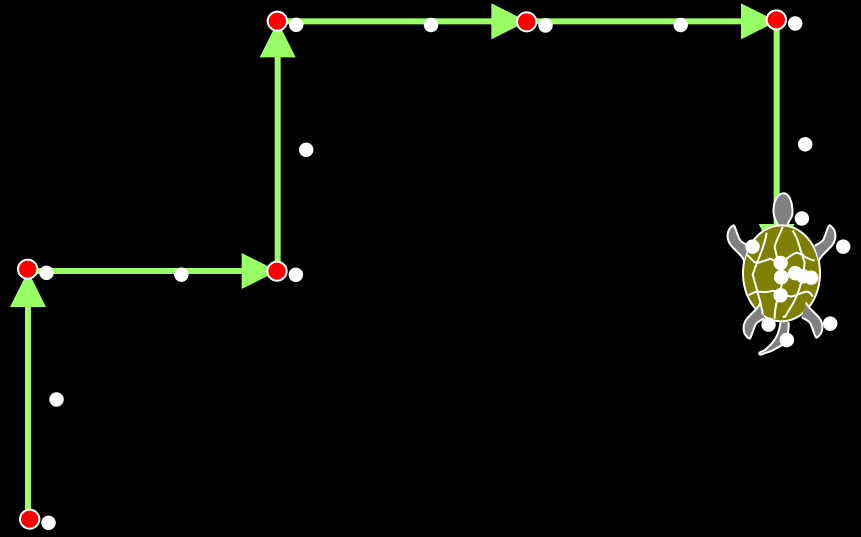

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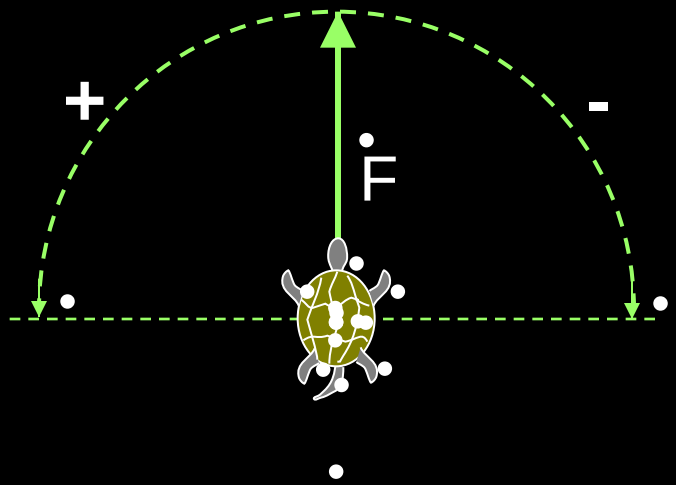

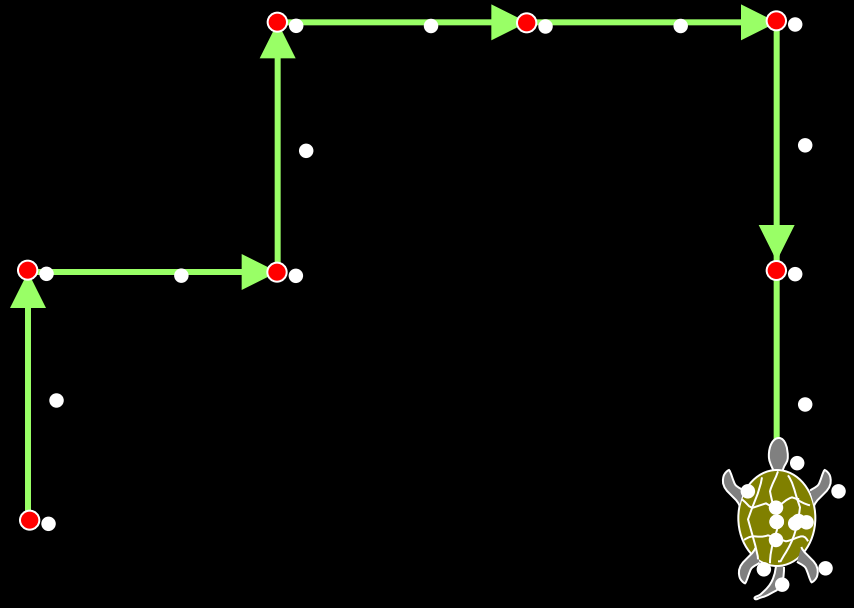

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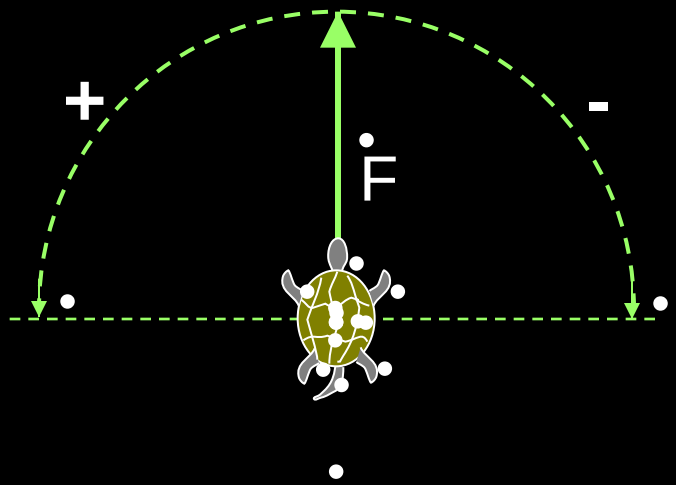

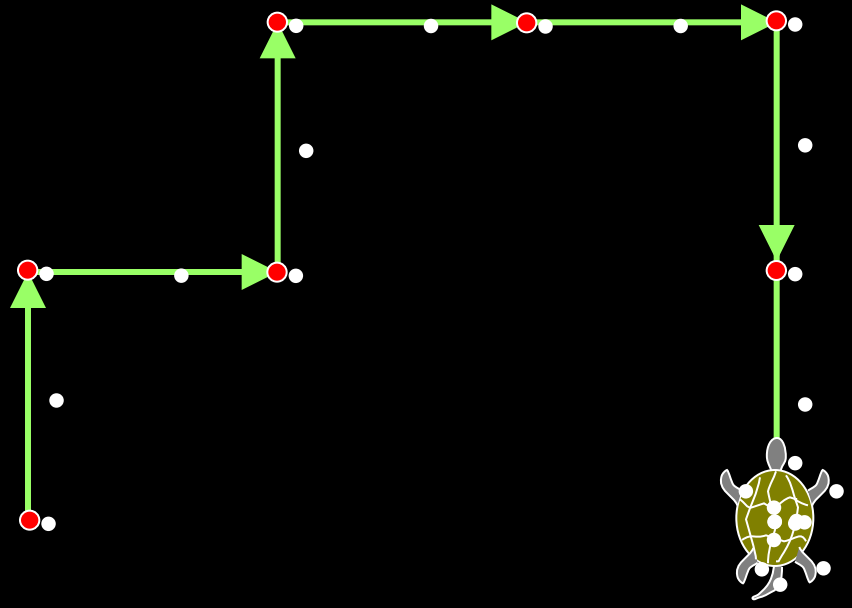

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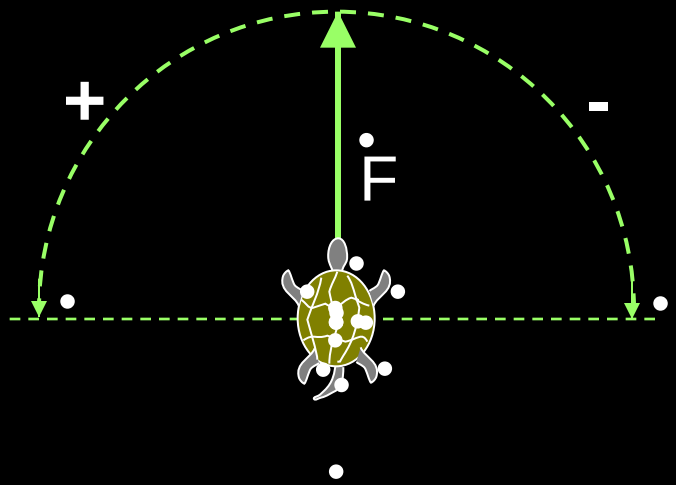

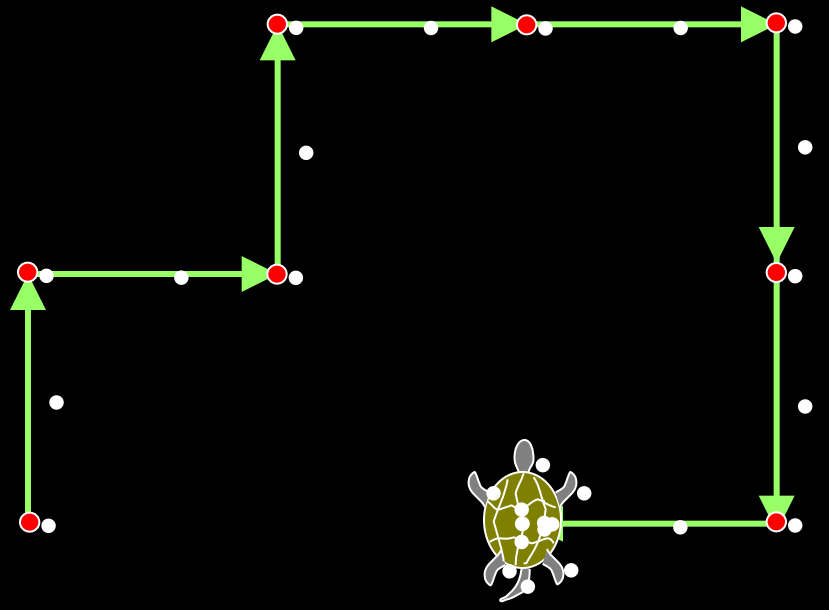

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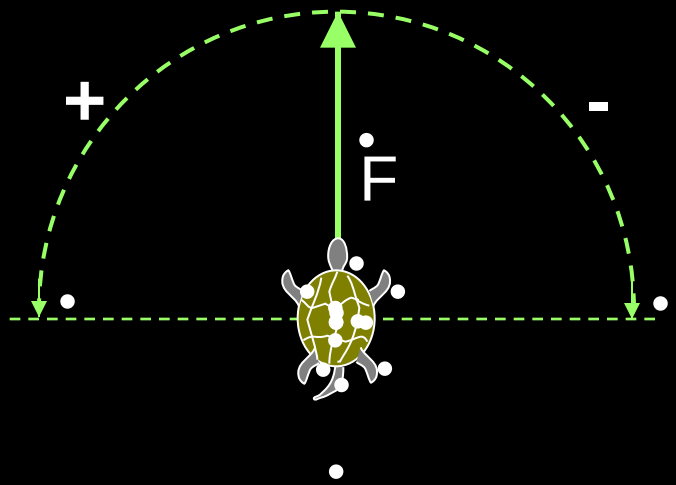

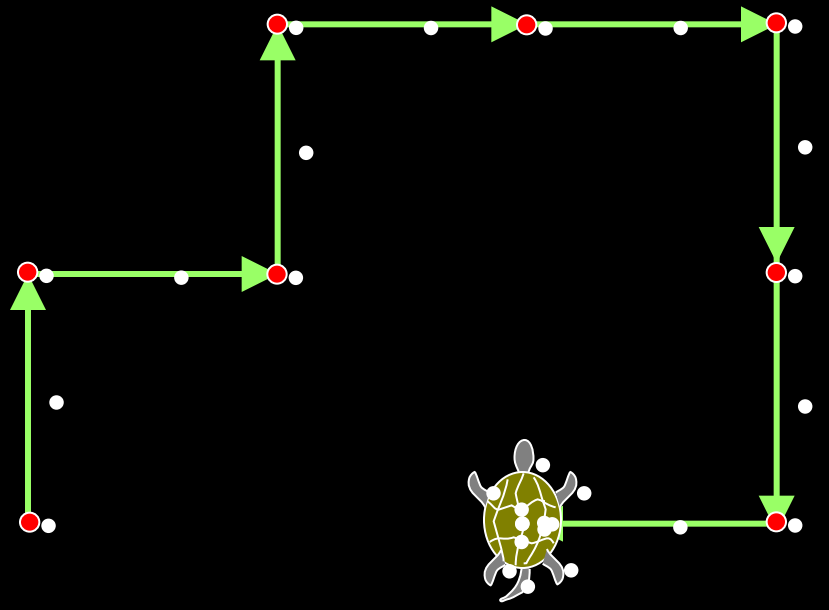

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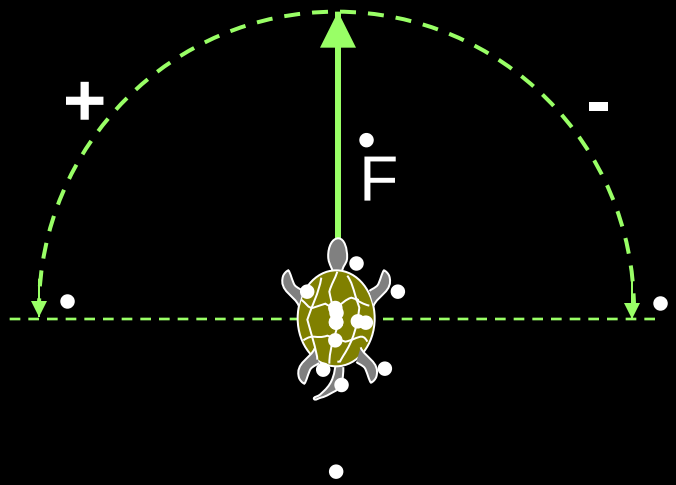
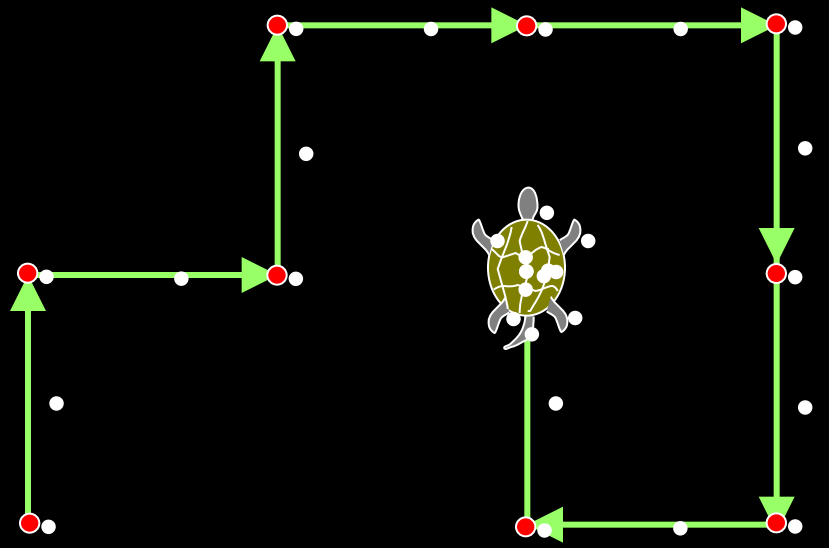
Turtle interpretation of strings





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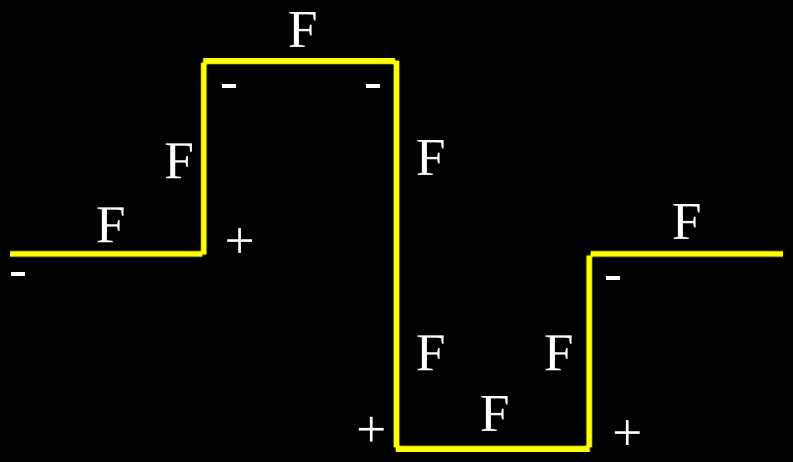
• Koch curve L-system



The Axiom:

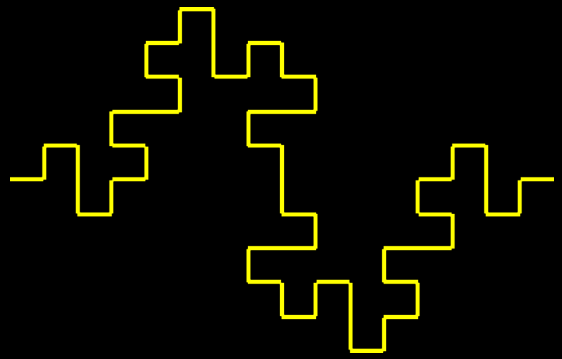
-F

Production, or re-writing rule:

F --> F+F-F-FF+F+F-F

After one step:

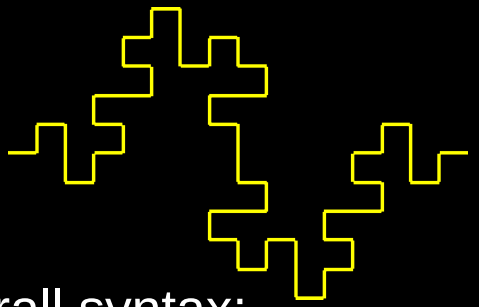
-F+F-F-FF+F+F-F



After two steps:

- F+F-F-FF+F+F-F + F+F-F-FF+F+F-F - F+F-F-FF+F+F-F
- F+F-F-FF+F+F-F F+F-F-FF+F+F-F + F+F-F-FF+F+F-F
+ F+F-F-FF+F+F-F - F+F-F-FF+F+F-F

• L-studio Koch curve model



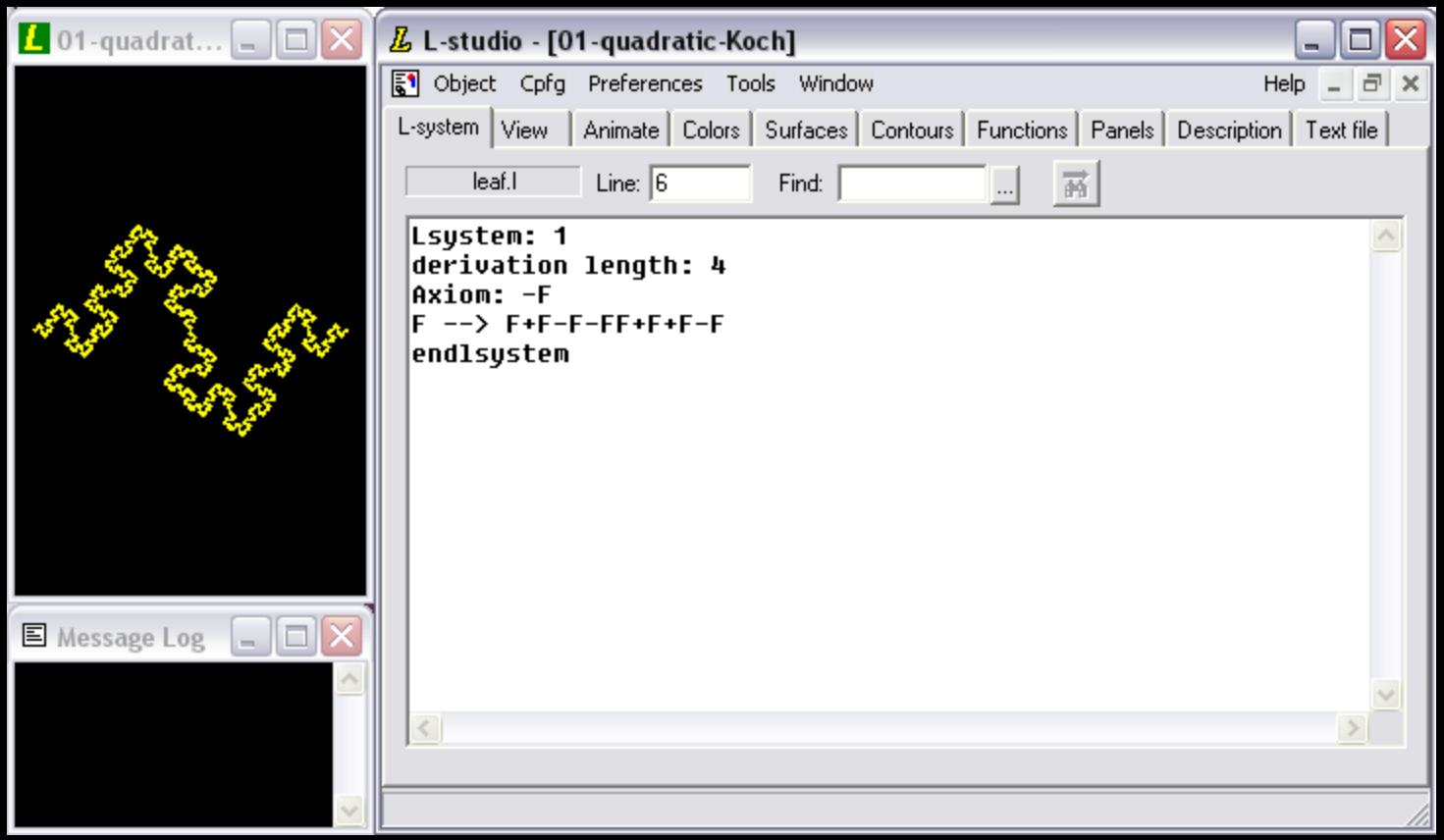
The L-System file has the following overall syntax:

Lsystem: 1 *mandatory opening, arbitrary number*
derivation length: 4 *the number of derivation steps*
Axiom: -F *the axiom*
F --> F+F-F-FF+F+F-F *one production*
endlsystem *mandatory closing*

Parameters, such as the magnitude of turns, are read from the view file. In this case, it has been defined as:

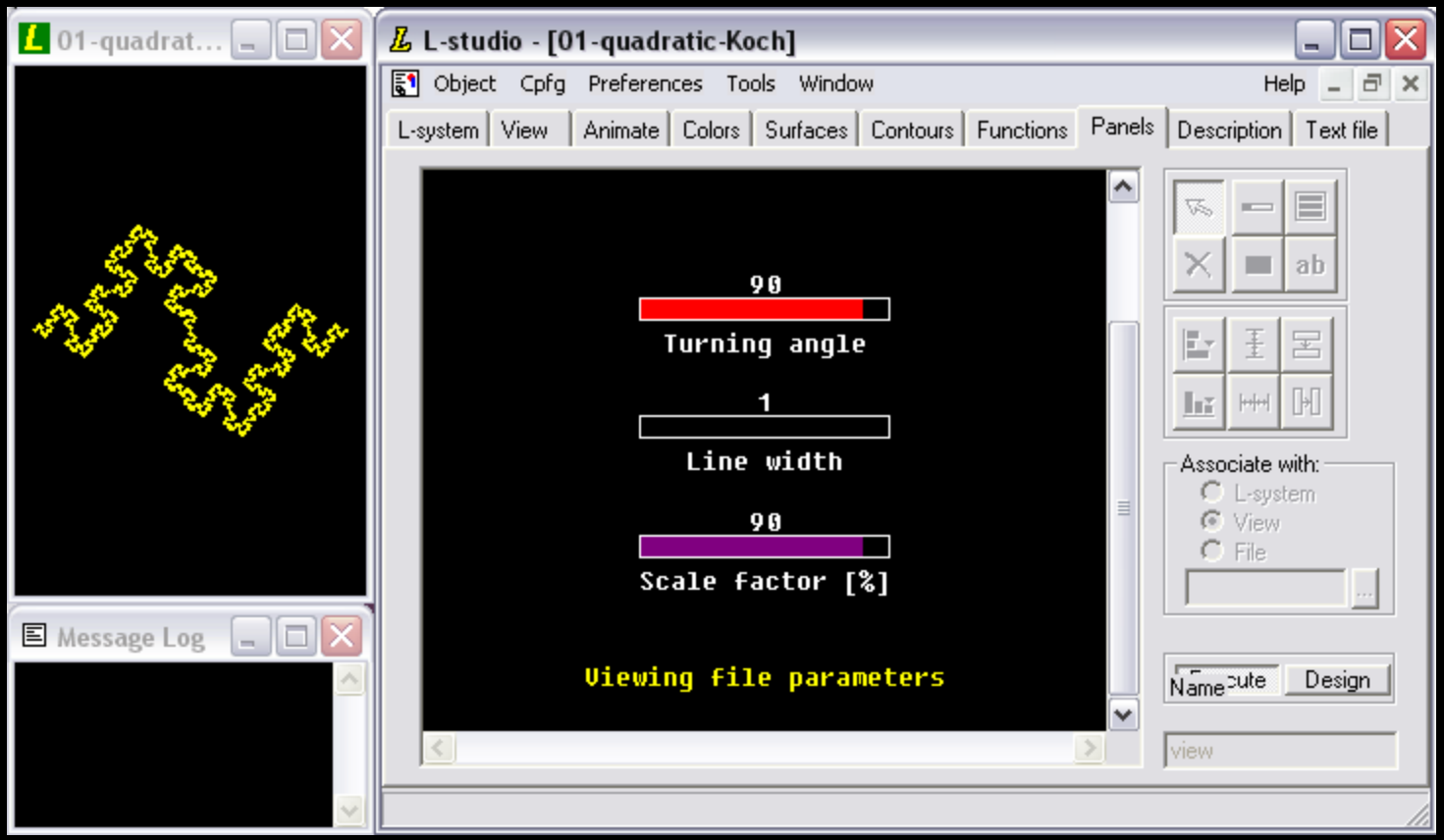
angle increment: 90 □□*the turning angle, in degrees*
initial line width: 1 pixels *width of line drawn*
initial color: 1 *index to the color map*
scale factor: 0.9000 *controls the size on the screen*

• Intro to L-studio



- Main view plus simulation window
- Different tabs show model parameter editors

• Intro to L-studio

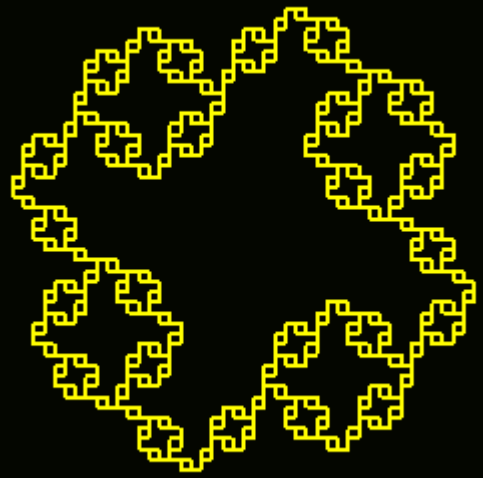


- Panels tab holds model parameters control sliders
- Visual, repeatable interface to model parameters



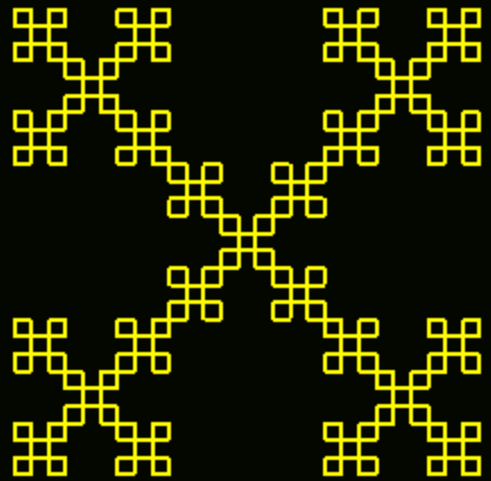
F-F-F-F

F → F+F-F-FF+F+F-F



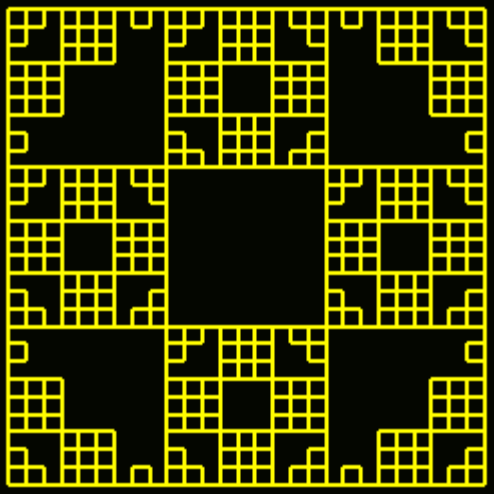
F-F-F-F

F → FF-F-F-F-F-F+F



F-F-F-F

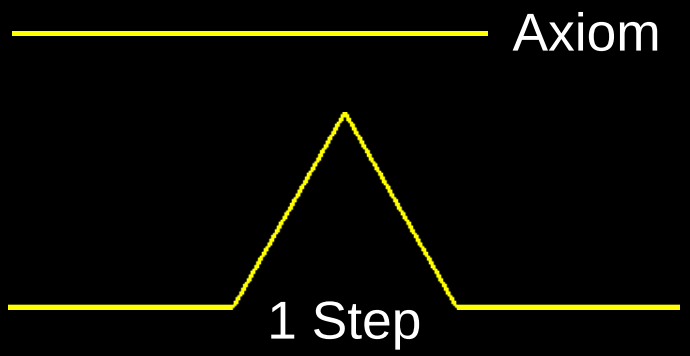
F → F-F+F+F-F

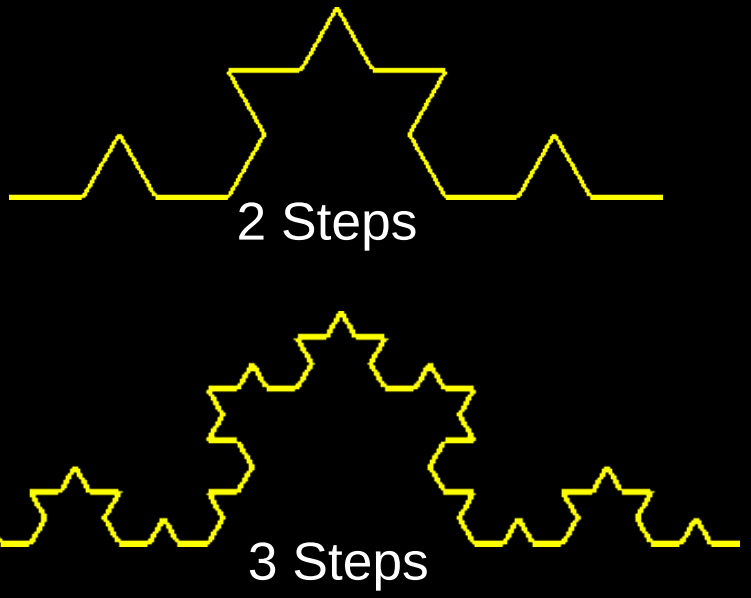


F-F-F-F

F → FF-F-F-F-FF

• Snowflake curve





L-System

- Lsystem: 1
- derivation length: 4
- Axiom: -(90)F *turn by 90 degrees (override)*
- F --> F+F--F+F
- endlsystem

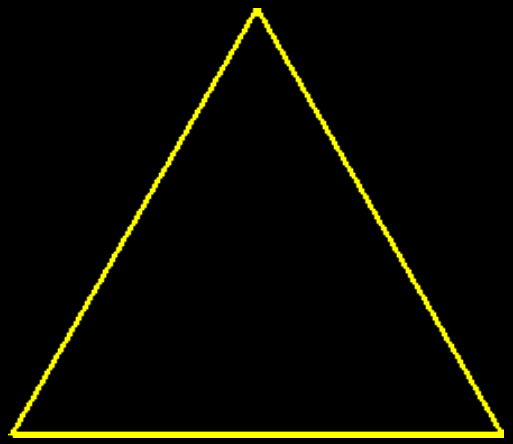
View file

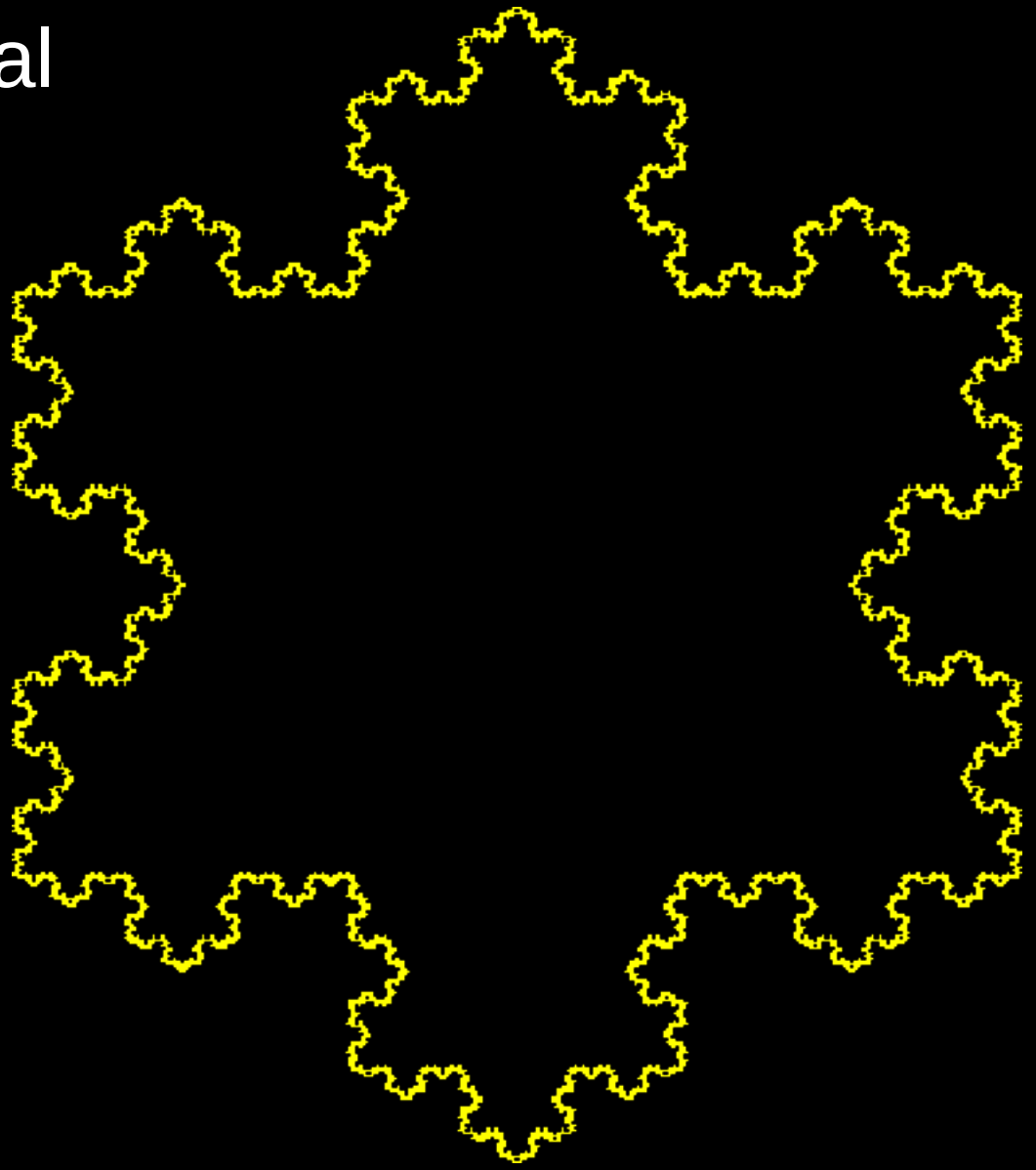
- angle increment: 60 *turns are now 60 degrees*
- initial line width: 1 pixels
- initial color: 1
- scale factor: 0.9000

- Snowflake fractal

Change the axiom to:

-(30)F--F--F



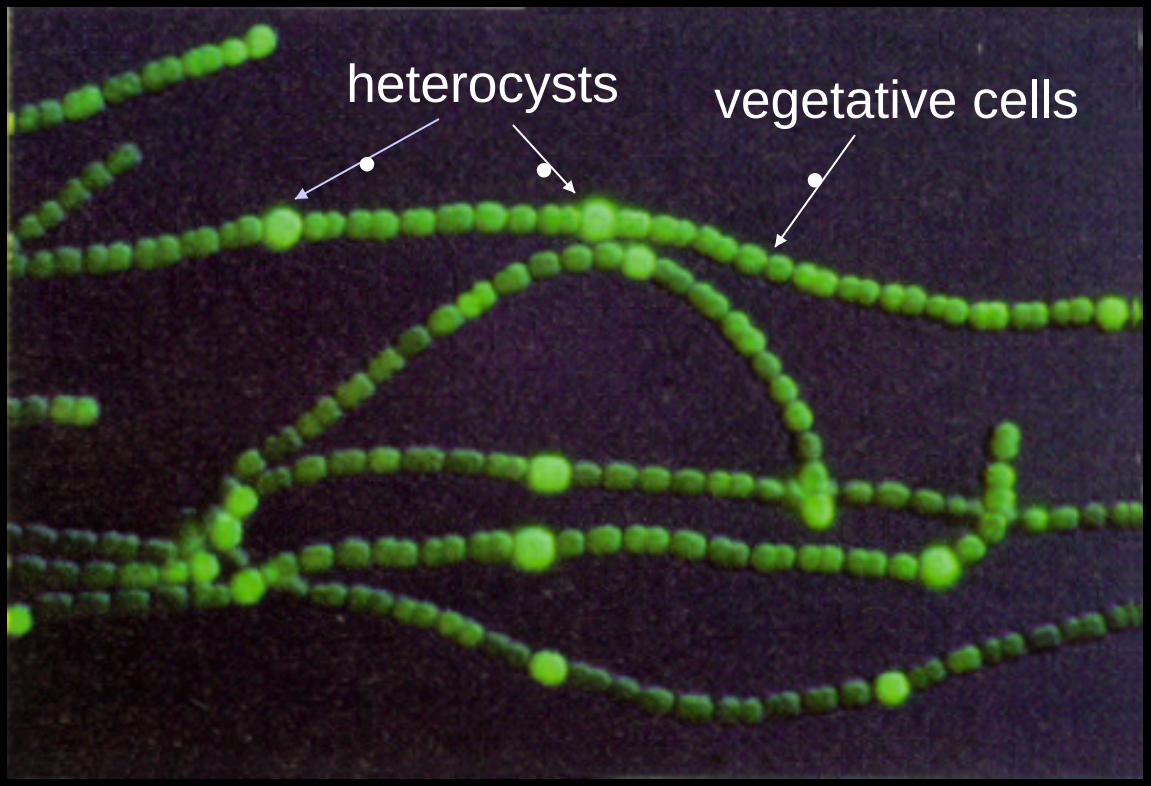


Do 5 steps:

F --> F+F--F+FF

02-SnowFlake

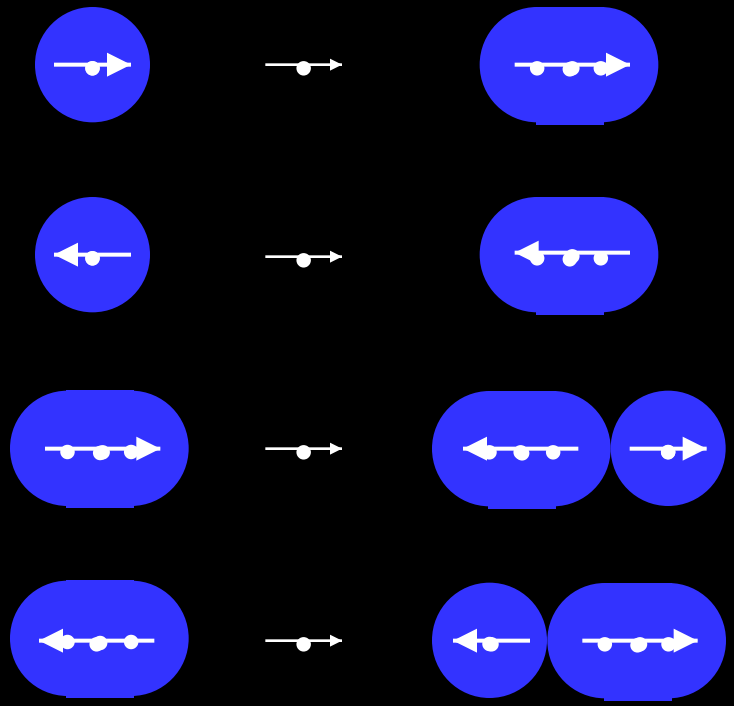
• Modeling Anabaena



Haselkorn, Science 282: 891-892, 1998

- Cyanobacteria in existence for 3.5 billion years
- Two incompatible processes, nitrogen fixation and photosynthesis
- Evolved a division of labor, with two types of cells
- Heterocysts, are able to fix nitrogen from the atmosphere
- Photosynthesis carried out in vegetative cells
- If distance between two heterocysts becomes too large, a vegetative cell will differentiate into a heterocyst

Vegetative segment development

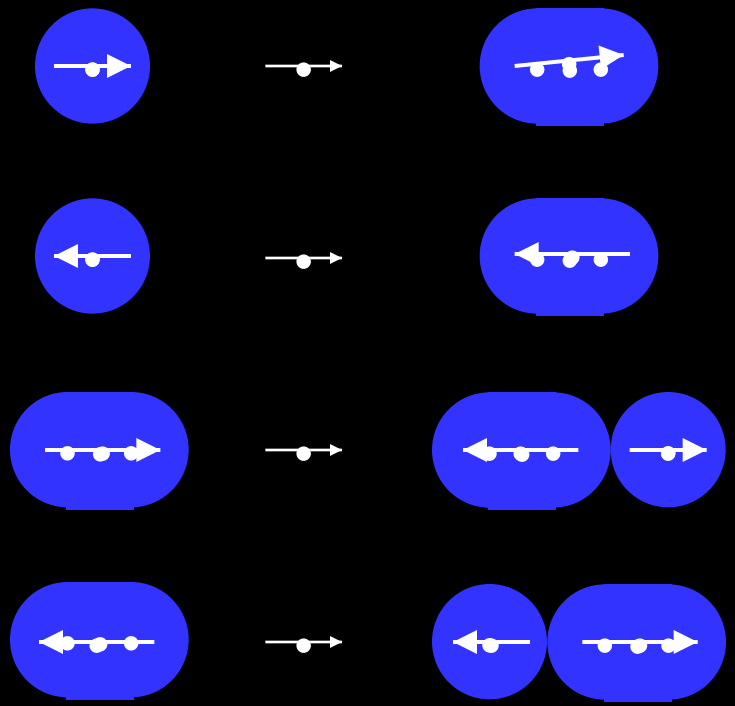


- Mitchison, Wilcox, 1972

L-system model

Alphabet:

$\vec{S}, \overleftarrow{S}, \vec{L}, \overleftarrow{L}$



- Cell division of Anabaena is asymmetric
- The shorter daughter cell has the same orientation as the parent, the longer one has opposite orientation
- The cells must reach a certain length before dividing

L-system model

Alphabet:

$\vec{S}$, $\overleftarrow{S}$, $\vec{L}$, $\overleftarrow{L}$

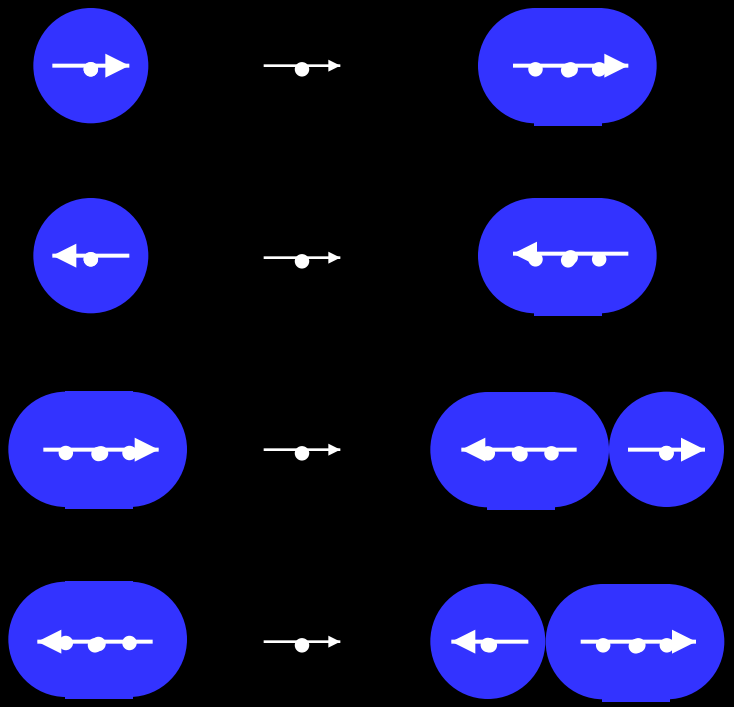
Productions:

$\vec{S} \rightarrow \vec{L}$

$\overleftarrow{S} \rightarrow \overleftarrow{L}$

$\vec{L} \rightarrow \vec{L}\ \vec{S}$

$\overleftarrow{L} \rightarrow \vec{S}\ \overleftarrow{L}$



L-system model

Alphabet:

$\vec{S}, \overleftarrow{S}, \vec{L}, \overleftarrow{L}$

Axiom:

$\vec{L}$

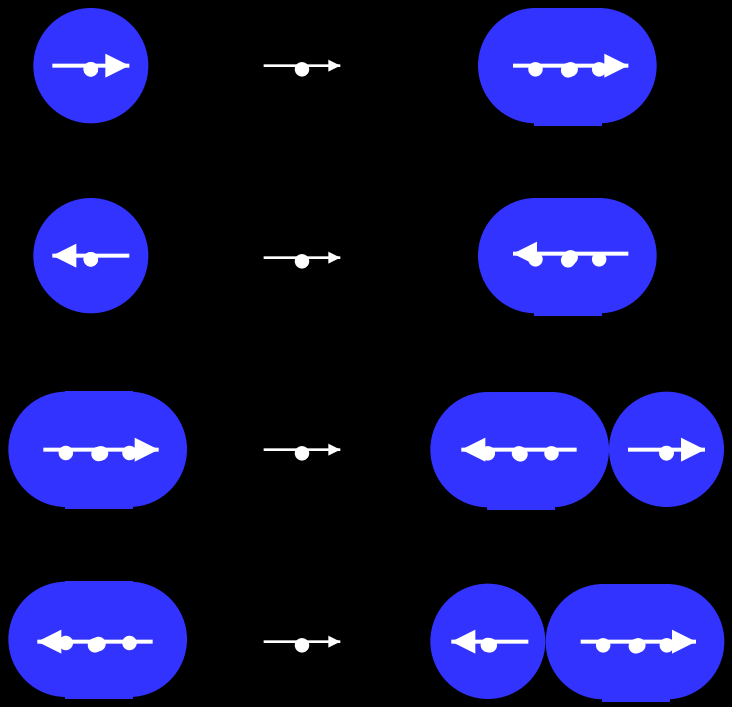
Productions:

$\vec{S} \rightarrow \vec{L}$

$\overleftarrow{S} \rightarrow \overleftarrow{L}$

$\vec{L} \rightarrow \vec{L}\,\overleftarrow{S}$

$\overleftarrow{L} \rightarrow \overleftarrow{S}\,\vec{L}$



L-system simulation

Alphabet:

$\vec{S}, \overleftarrow{S}, \vec{L}, \overleftarrow{L}$

Axiom:

$\vec{L}$

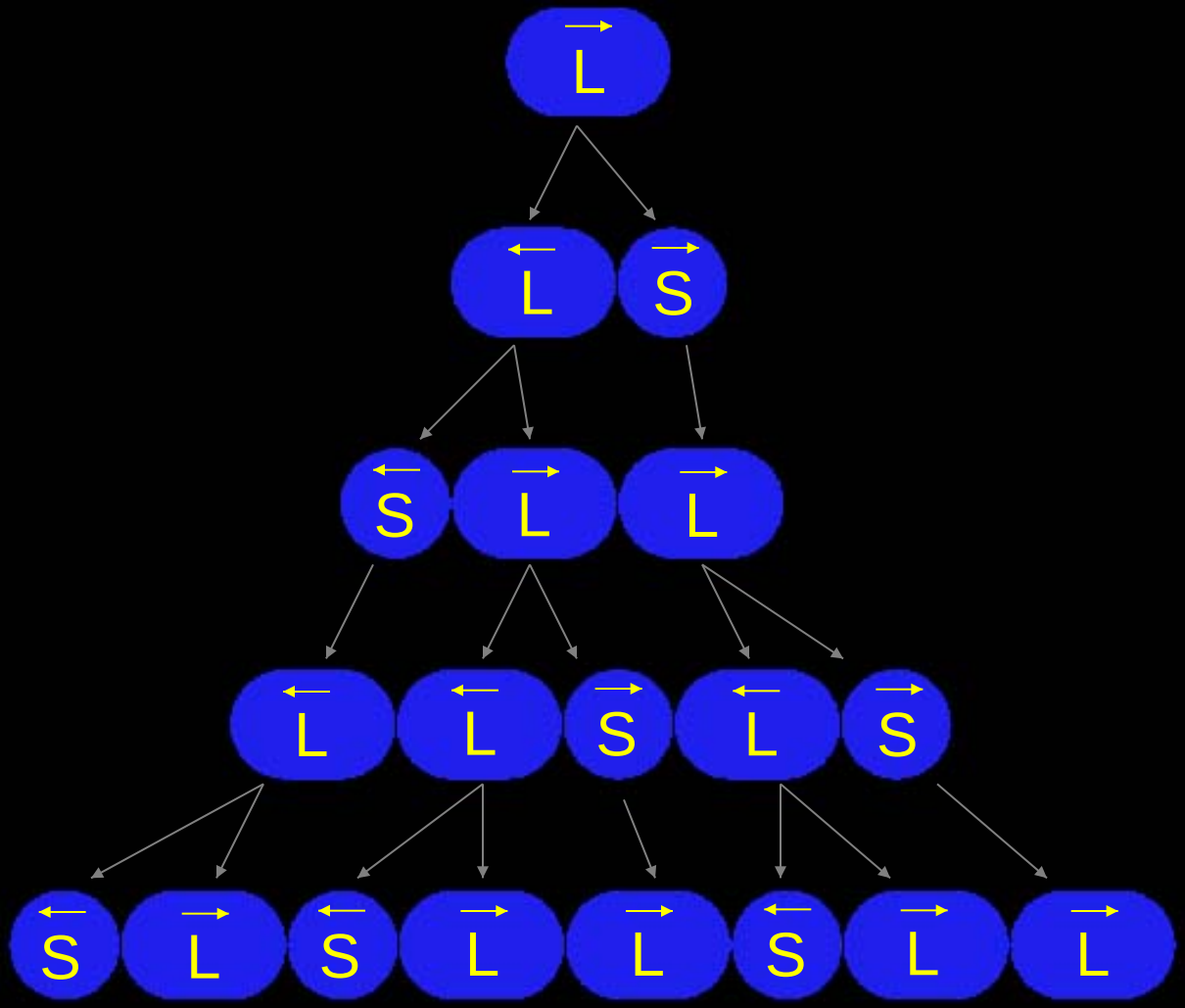
Productions:

$\vec{S} \rightarrow \vec{L}$

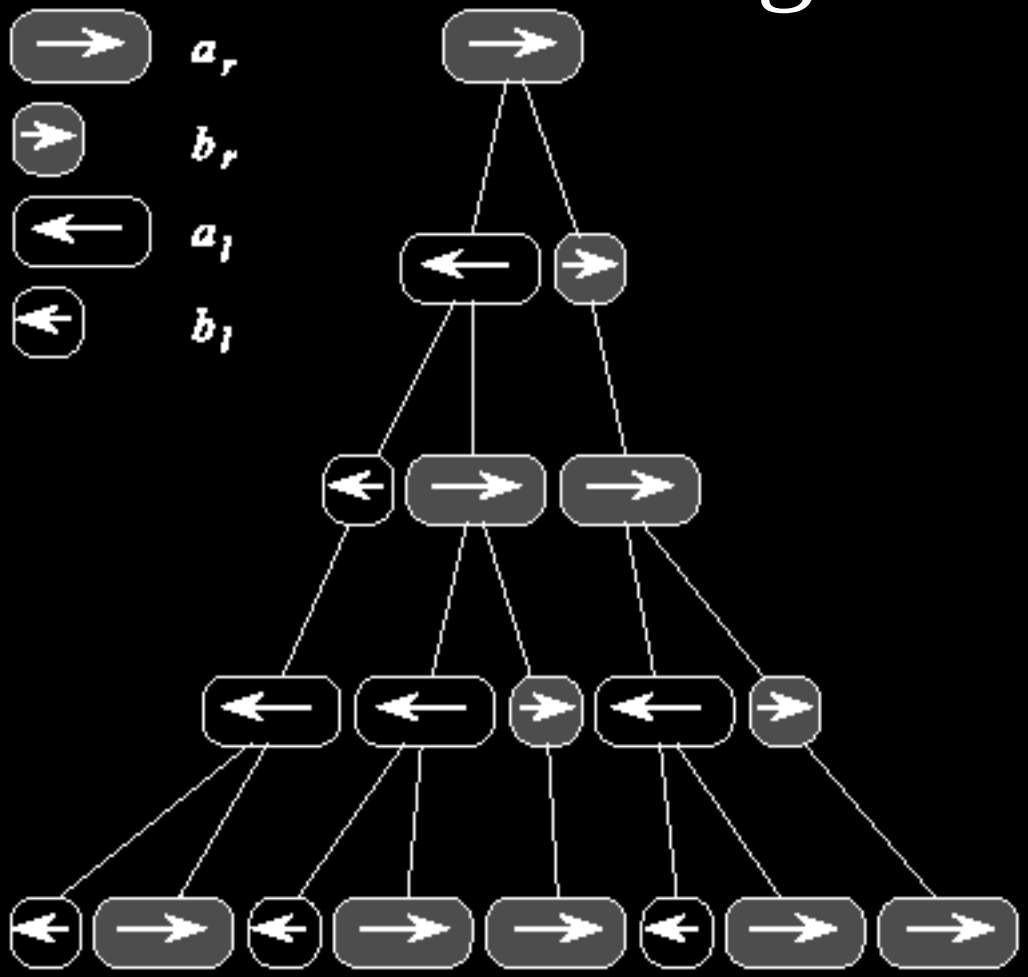
$\overleftarrow{S} \rightarrow \overleftarrow{L}$

$\vec{L} \rightarrow \overleftarrow{L}\,\vec{S}$

$\overleftarrow{L} \rightarrow \overleftarrow{S}\,\overleftarrow{L}$



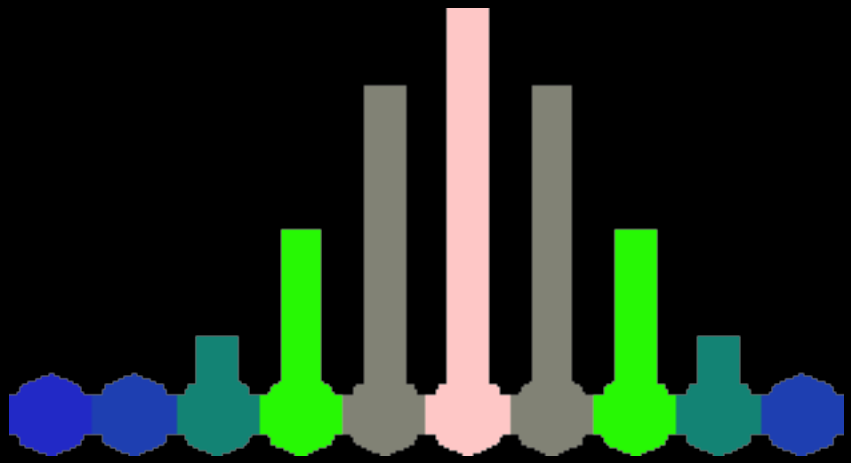
• Modeling Anabaena



ω : a_r
p_1 : $a_r \rightarrow a_l b_r$
p_2 : $a_l \rightarrow b_l a_r$
p_3 : $b_r \rightarrow a_r$
p_4 : $b_l \rightarrow a_l$

a_r
$a_l b_r$
$b_l a_r a_r$
$a_l a_l b_r a_l b_r$
$b_l a_r b_l a_r a_r b_l a_r a_r$
$\ldots$

• Modeling Diffusion on a Line of Cells

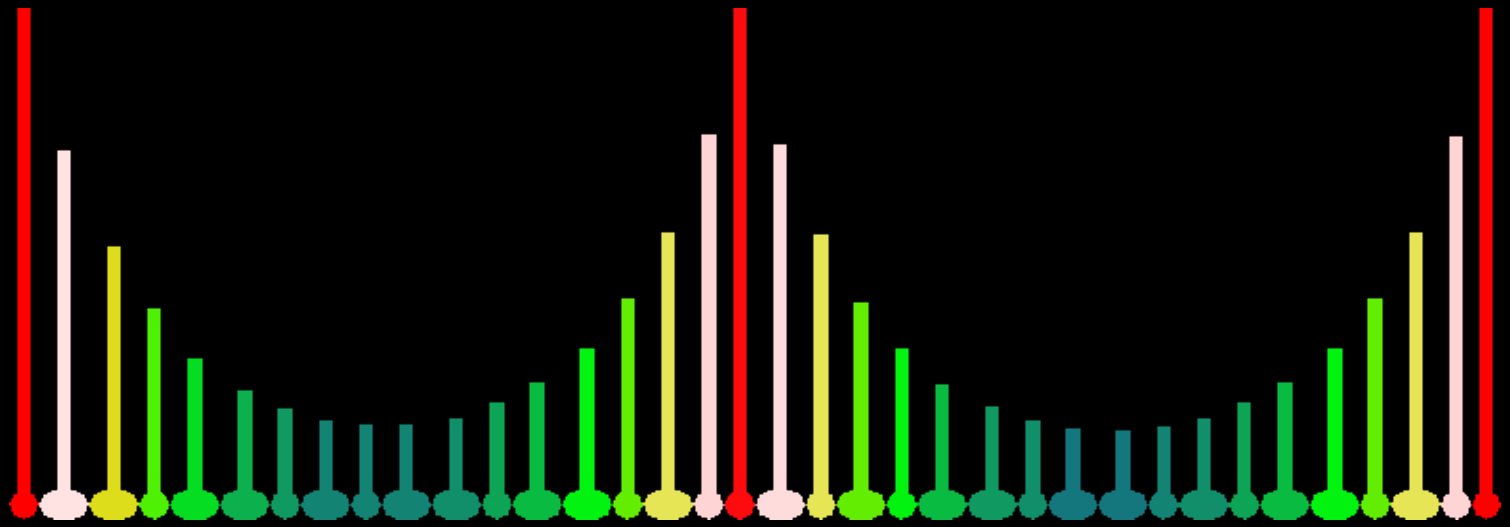


Given a module $C(x)$, that represents a cell with a substance of concentration x, diffusion can be modeled by the following production:

$$C(cl) < C(c) > C(cr) \dashrightarrow C(c + (cl - c + cr - c) * D)$$

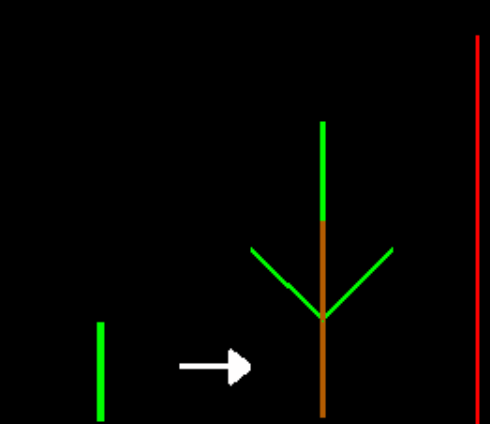
Speed of diffusion will depend on diffusion constant D

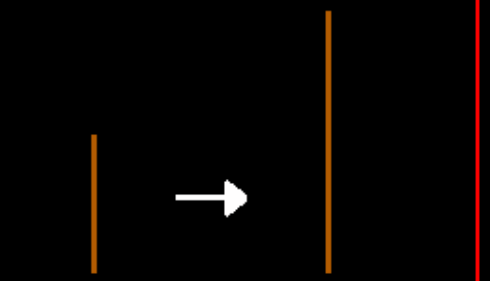
• Modeling Anabaena



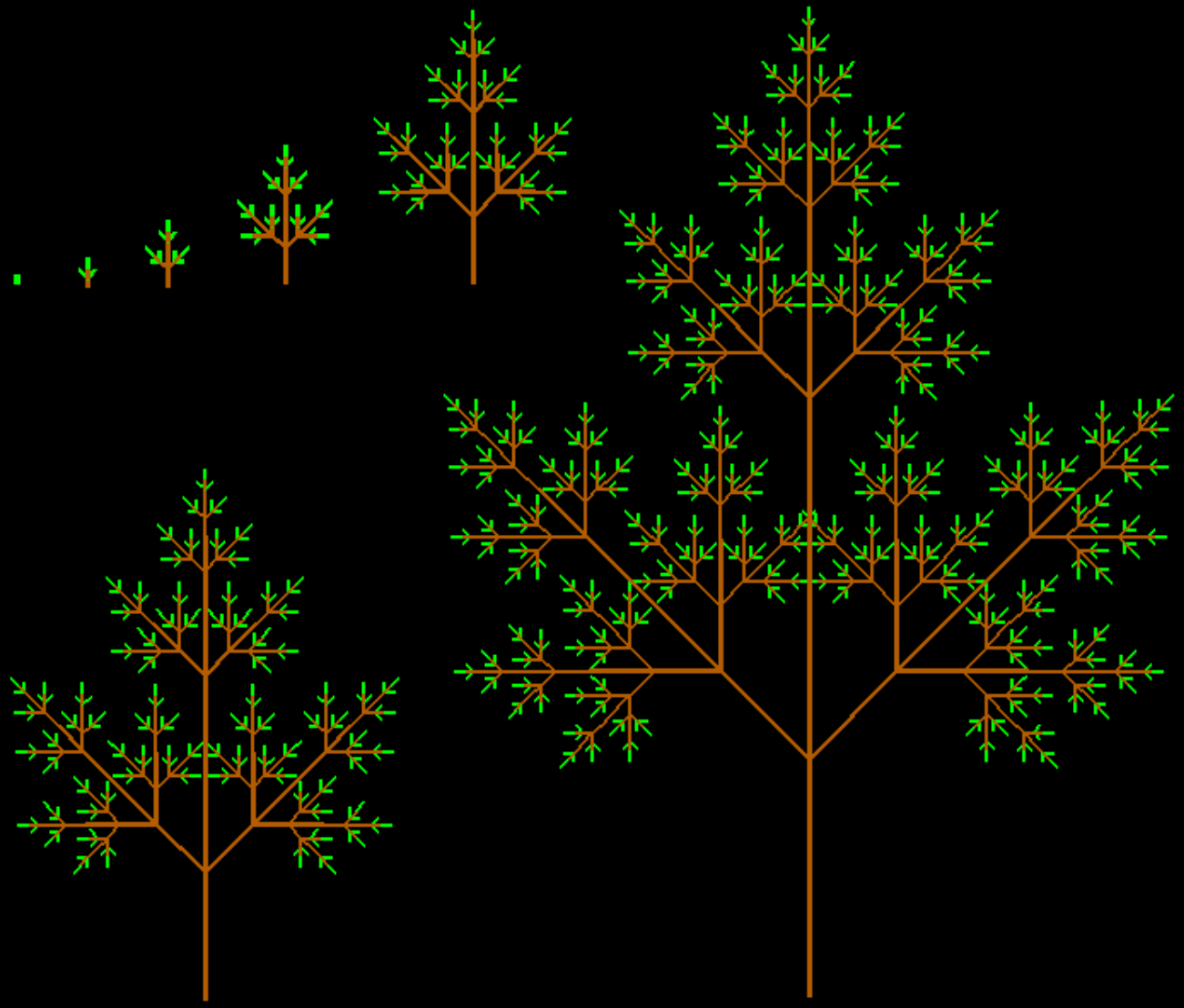
- Heterocysts give off an inhibitor that diffuses to other cells
- When inhibition falls below a threshold, cells differentiate into heterocysts

Growing plant as a dynamic system with a dynamic structure

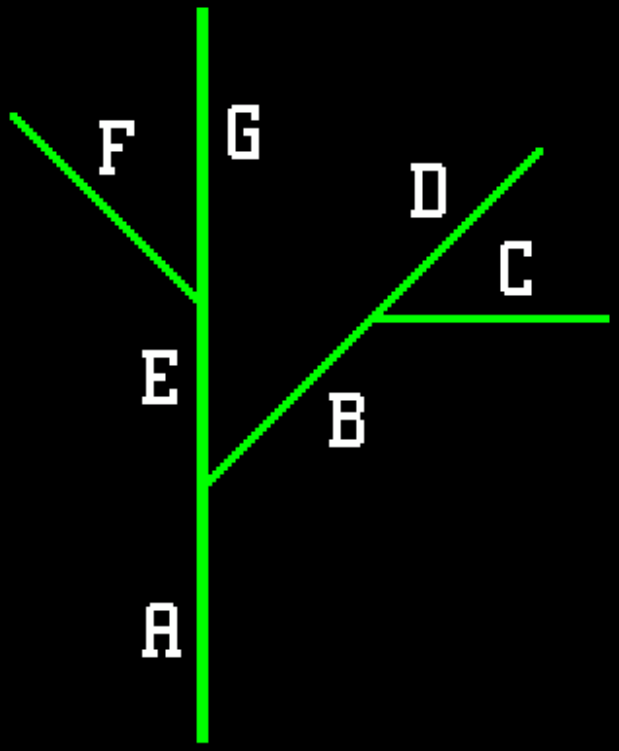




productions

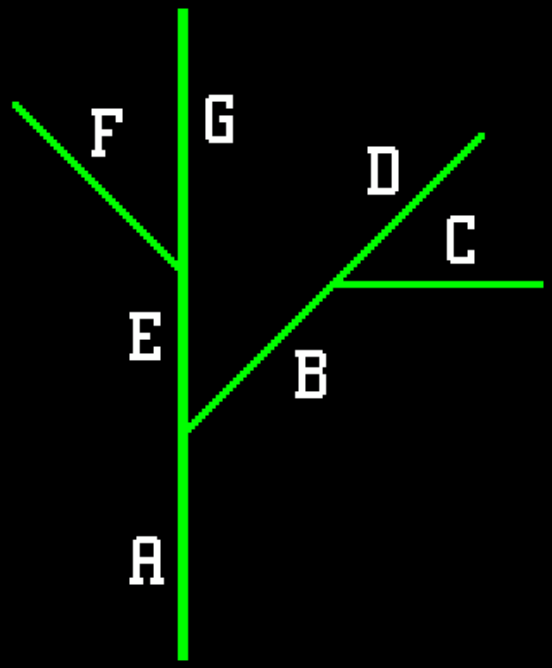


Bracketed string representation of an axial tree



A [B [C] D] E [F] G

Toward geometric interpretation of bracketed strings

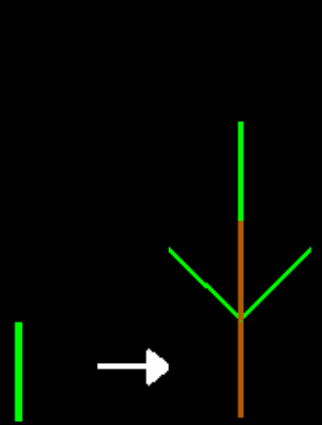


A [– B [– C] D] E [+F] G

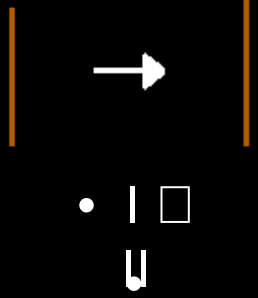
Matching pairs of brackets enclose branches

+ and – indicate directions

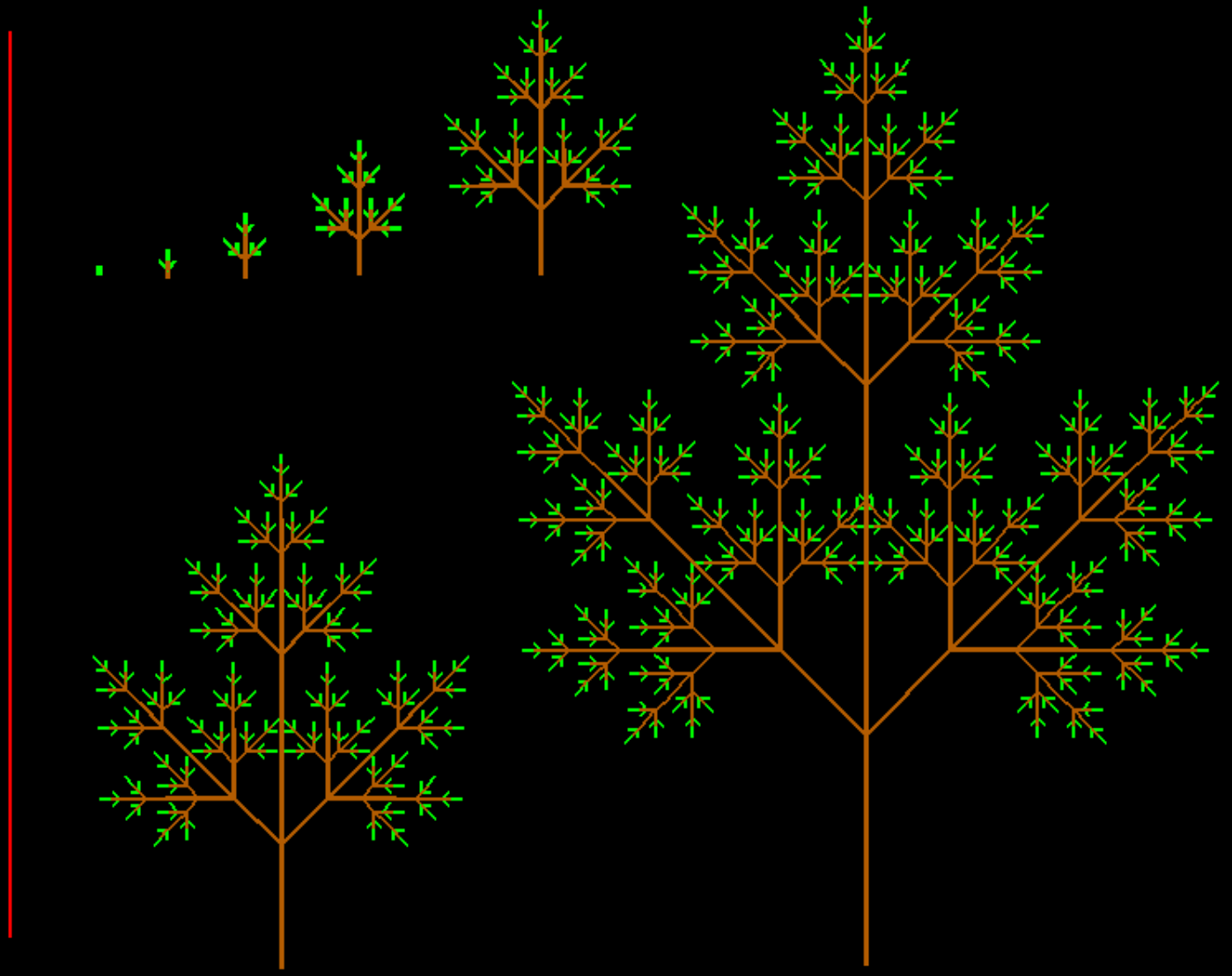
Textual specification of productions



- A → I[A][A]IA



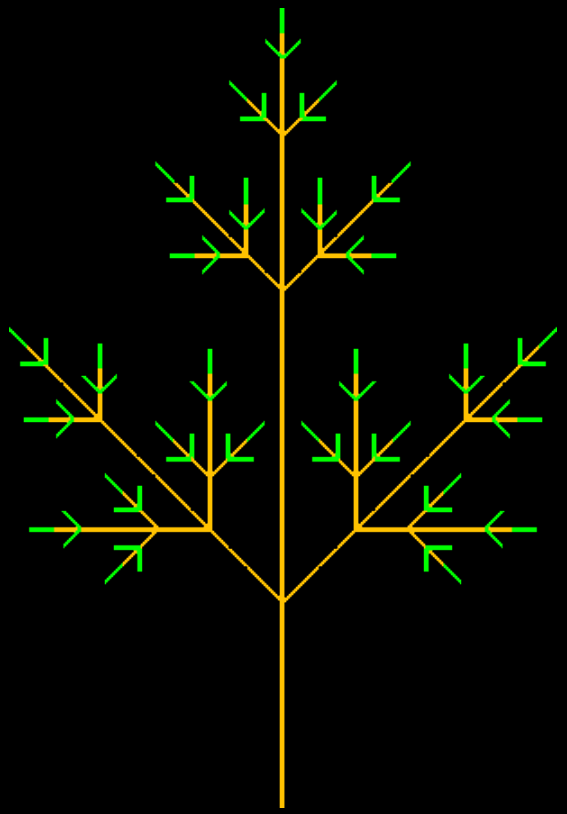
- I → I



• Compound Branching Structure

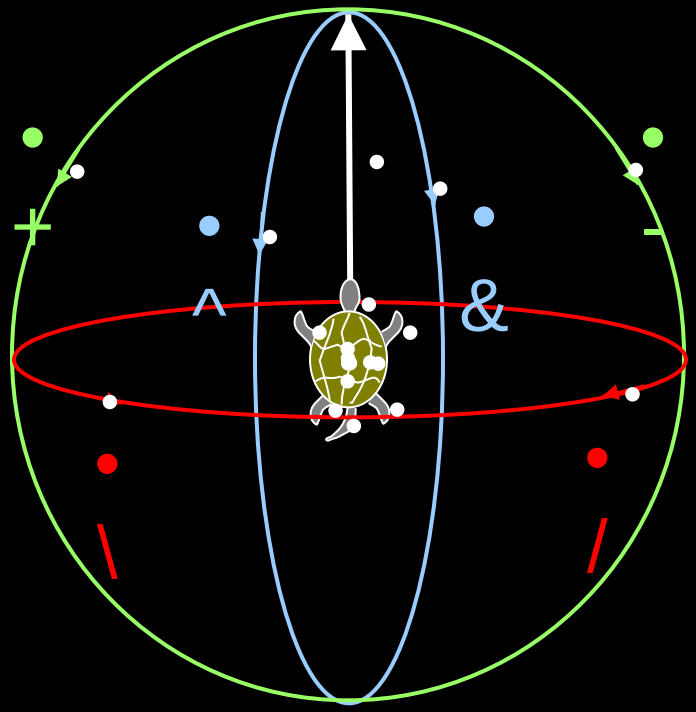
Axiom: A
A --> I[+A][-A]IA
I --> II
homomorphism
A --> ;F
I --> F
endlsystem

angle increment: 45

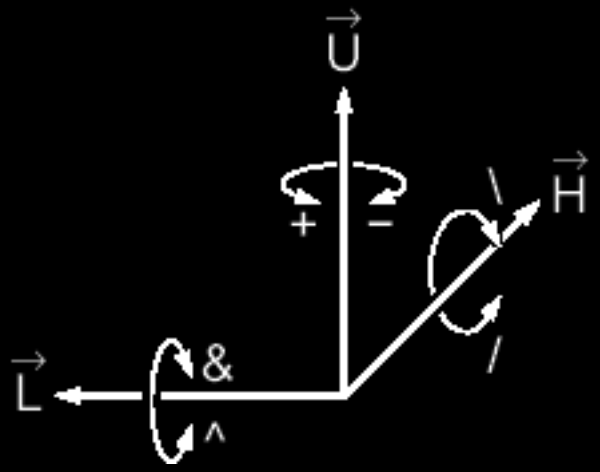


- With a couple of rules we can create a simple branching structure
- The symbols “I” and “A” have no graphical interpretation
- We use interpretation rules (homomorphism) to tell cpfg how to draw them
- More readable A-Apex, I-Internode
- Symbols “[“ and “]” will save and restore turtle position

Manipulating the turtle in 3D



• 3D Turtle commands

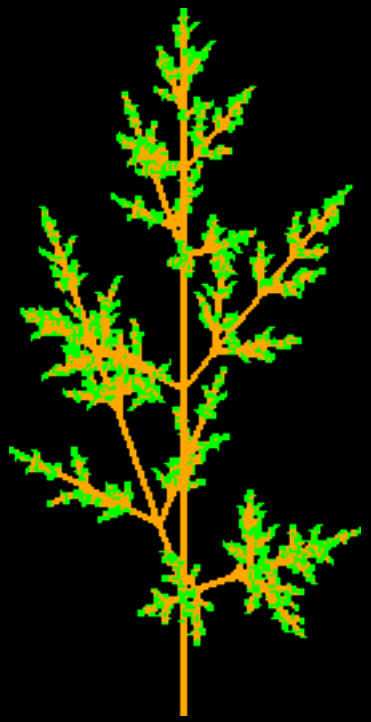


+ Turn left around the U axis
- Turn right around the U axis
& Pitch down around the L axis
^ Pitch up by around the L axis
\ Roll left by around the H axis
/ Roll right by around the H axis

The turtle can be rotated around its heading (H), left (L), or, up (U) vector

• 3D Branching

So far all of the models have been planar. By adding a rotate command, we can create a 3 dimensional decussate pattern:



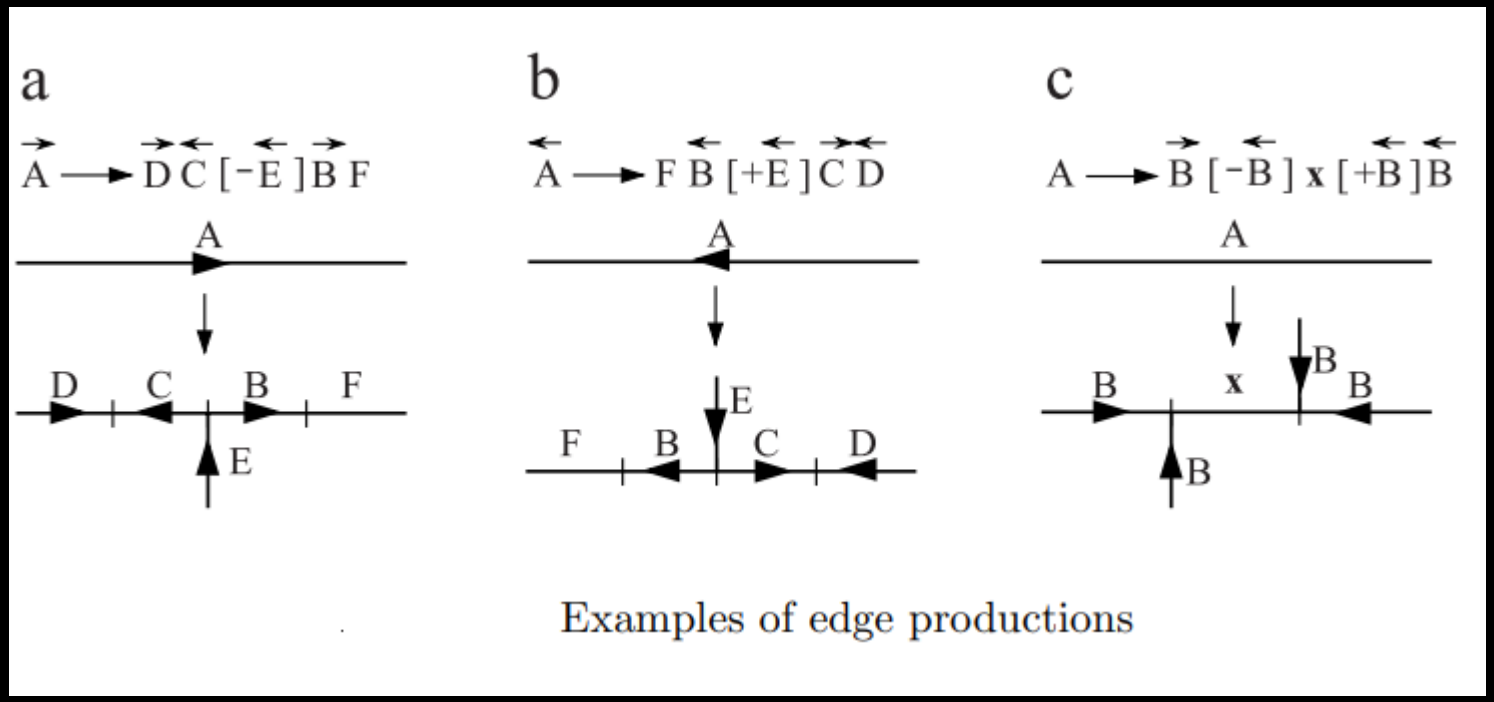
Lsystem: 1
derivation length: STEPS
Axiom: A(0)
A(t) : t < DELAY --> A(t + dt)
A(t) : t >= DELAY --> /(90)M(t)A(t/2.0)
I(x) --> I(x * RATE ^ dt)

•
•
•

The “/” command will rotate the turtle by the default angle Here we have specified 90°, creating decussate phyllotaxis

• Map L-systems

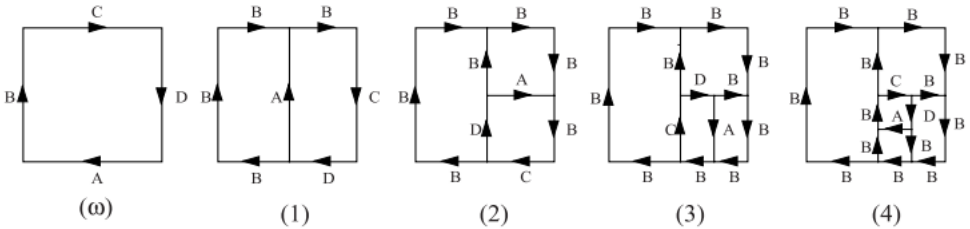
- Complex topology – Graphs with cycles
- Map L-systems – Proposed by Lindenmayer and Rozenberg.
- A Map is composed of regions.
- Boundaries of regions are composed of edges, that meets in vertices.
- Regions – cells and edges – cell walls.



Examples of edge productions

Derivation Step in Map L-Systems

- A derivation step consists of two phases
  - Each edge in the map is replaced by successor edges and markers using the corresponding edge production
  - Each region is scanned for matching markers
- Two markers are considered matching if:
  - They appear in the same region.
  - They have the same label, and
  - One marker is directed away from the its incident edge while the other is directed towards the edge, or both markers are neutral



ω : $\vec{A}\vec{B}\vec{C}\vec{D}$

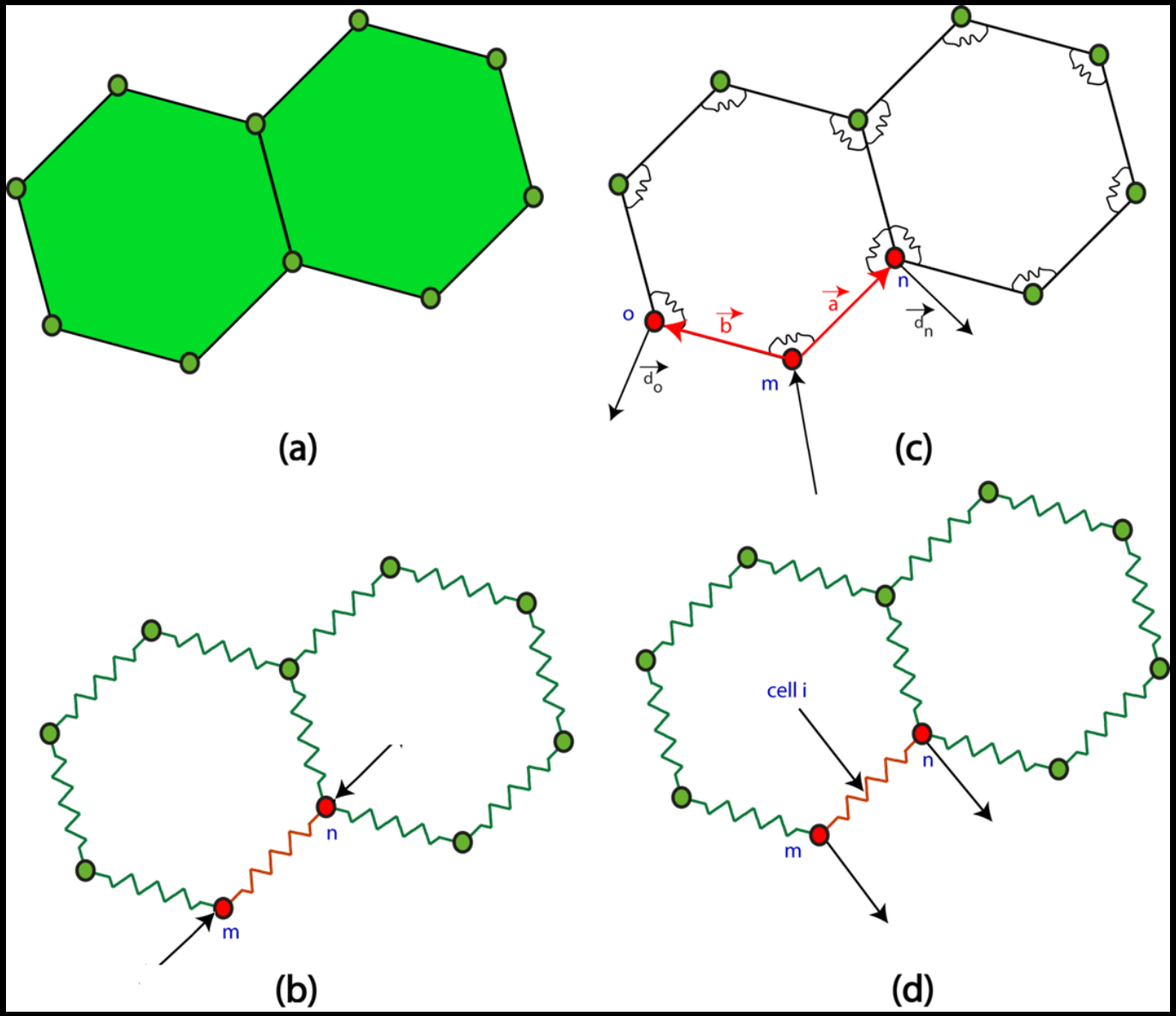
p_1 : $\vec{A} \rightarrow \vec{D}[-\vec{A}]\vec{B}$

p_2 : $\vec{B} \rightarrow \vec{B}$

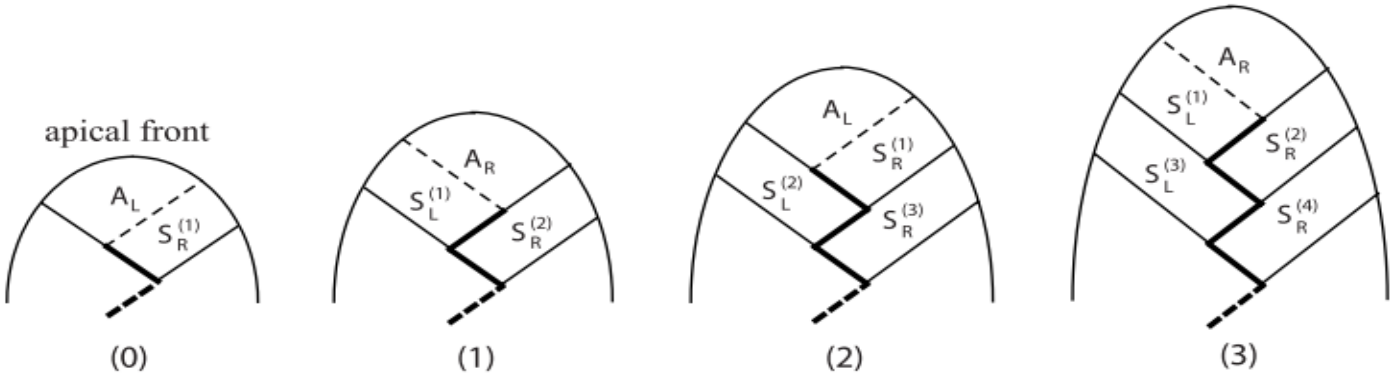
p_3 : $\vec{C} \rightarrow \vec{B}[-\overleftarrow{A}]\vec{B}$

p_4 : $\vec{D} \rightarrow \vec{C}$

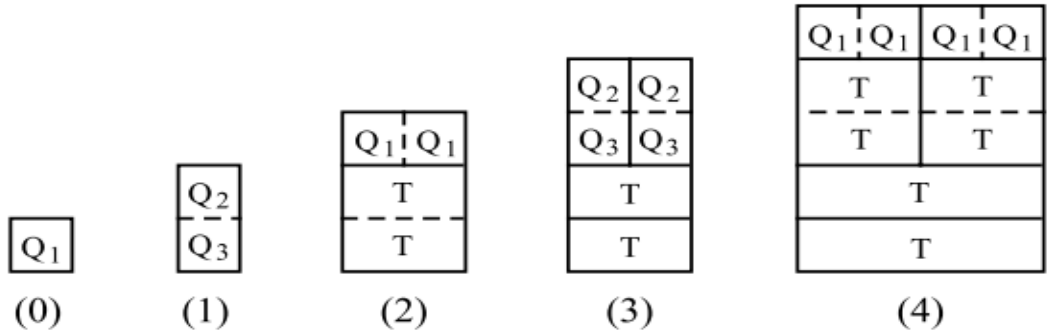
- Vertex-based mechanical model



- Development Sequence of Apical and Basal Segments in *Microsorium*

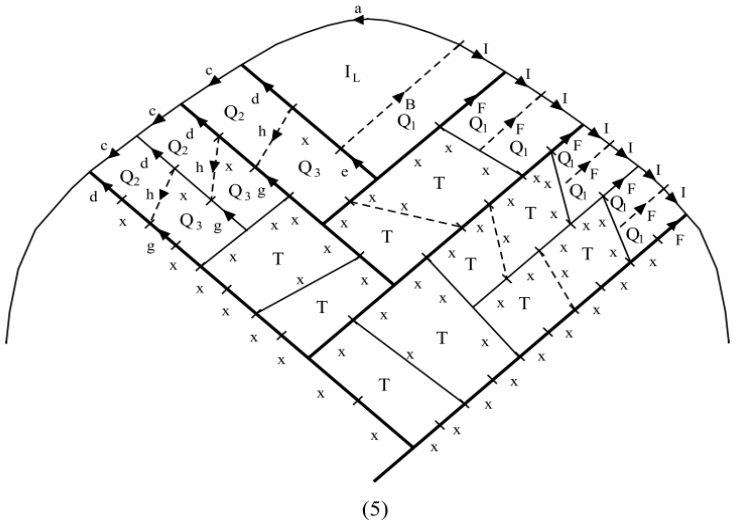


Apical production of segments. The labels A_R and A_L denote apical cells producing right segment S_R and left segment S_L, respectively. Dashed lines indicate the newly created division wall. The superscripts represent segment age. The internal structure of segments is not shown.



Developmental sequence of a *Microsorium* segment

- Map L-Systems rules for *Microsorium Development*



(5)

ω : $\vec{A}\overleftarrow{D}x\vec{b}$

l_1 : $\vec{a} \rightarrow \overleftarrow{A}\,[+\,\overleftarrow{b}]\,\vec{i}$

l_2 : $\vec{b} \rightarrow \vec{e}\,[-\,\vec{B}]\,x\,[+\,\vec{h}]\,\vec{d}$

l_3 : $\vec{d} \rightarrow \vec{f}$

l_4 : $\vec{f} \rightarrow \vec{g}\,[-\,\overleftarrow{h}]\,x\,[+\,\vec{h}]\,\vec{d}$

l_5 : $\vec{h} \rightarrow x\,[-\,\vec{f}]\,x$

l_6 : $\vec{i} \rightarrow \vec{c}$

l_7 : $\vec{c} \rightarrow \vec{i}\,[+\,\overleftarrow{f}]\,\vec{i}$

l_8 : $\vec{e} \rightarrow x\,[+\,x]\,x$

l_9 : $\vec{g} \rightarrow x\,[-\,x]\,x\,[+\,x]\,x$

r_1 : $\vec{A} \rightarrow \overleftarrow{a}\,[-\,\overleftarrow{B}]\,\vec{I}$

r_2 : $\vec{B} \rightarrow \vec{E}\,[+\,\vec{b}]\,x\,[-\,\vec{H}]\,\vec{D}$

r_3 : $\vec{D} \rightarrow \vec{F}$

r_4 : $\vec{F} \rightarrow \vec{G}\,[+\,\overleftarrow{H}]\,x\,[-\,\vec{H}]\,\vec{D}$

r_5 : $\vec{H} \rightarrow x\,[+\,\vec{F}]\,x$

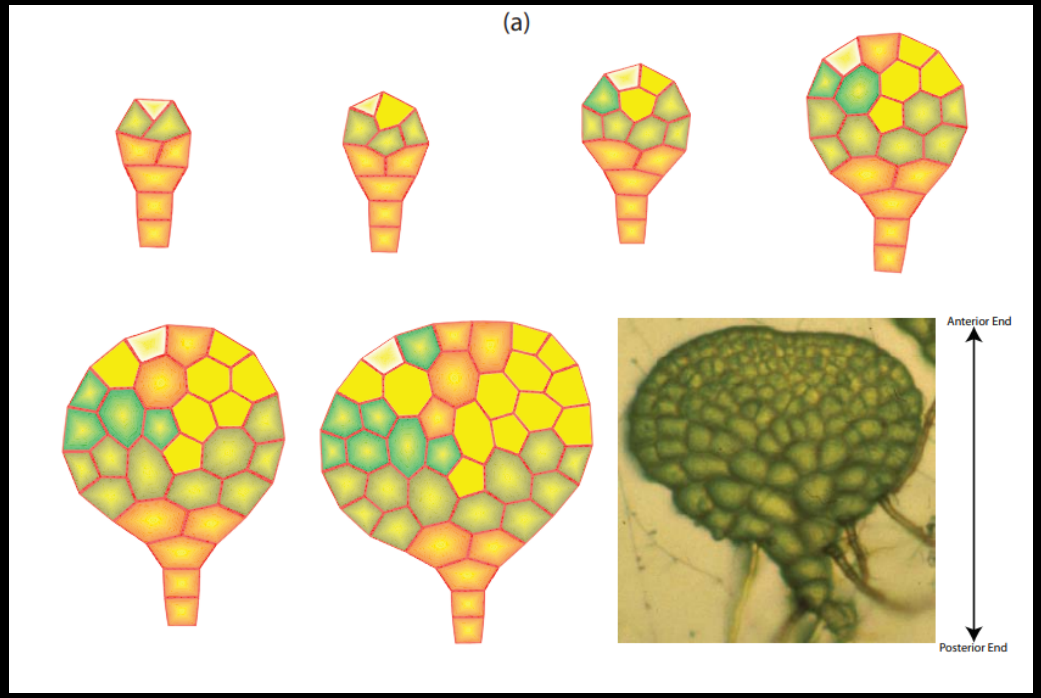
r_6 : $\vec{I} \rightarrow \vec{C}$

r_7 : $\vec{C} \rightarrow \vec{I}\,[-\,\overleftarrow{F}]\,\vec{I}$

r_8 : $\vec{E} \rightarrow x\,[-\,x]\,x$

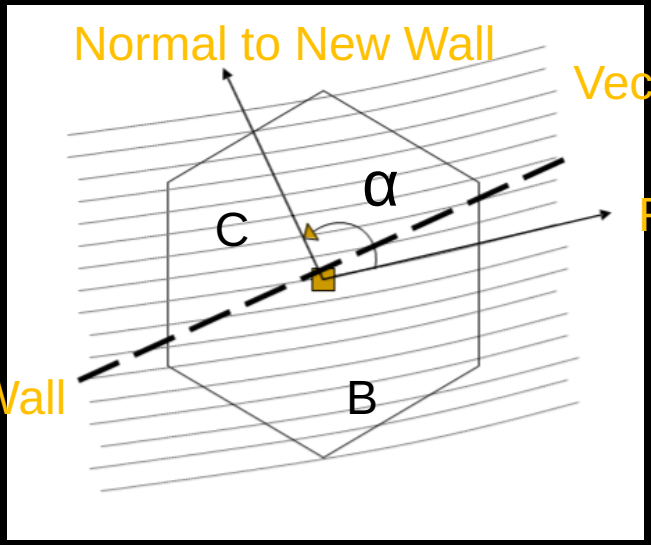
r_9 : $\vec{G} \rightarrow x\,[+\,x]\,x\,[-\,x]\,x$

Developmental sequence of *Microsorium*



• Cell Systems

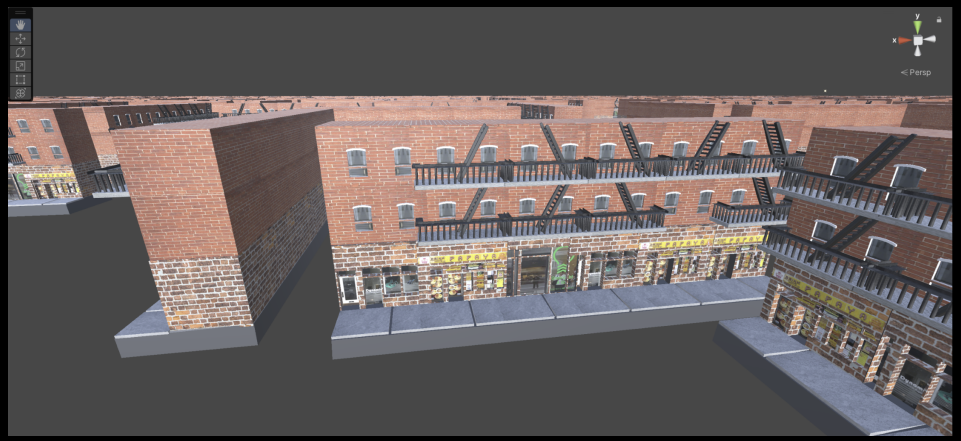
- More straightforward way – Pruskiewicz et. al., (1992).
- A→B
  - Change the cell label to A to B) (Not edge productions).
- A→B↑(α,r)C
  - Spilts cell A into two cells labeled B and C.
  - α – angle between the direction of the reference vector measured at the centre of gravity before division and the normal to the wall.
  - Cell B lies on the side of the wall opposite to the direction of normal and while C lies on the side pointed to by the normal.
  - The areas of the cells B is r and C is 1-r of the parent cell A.

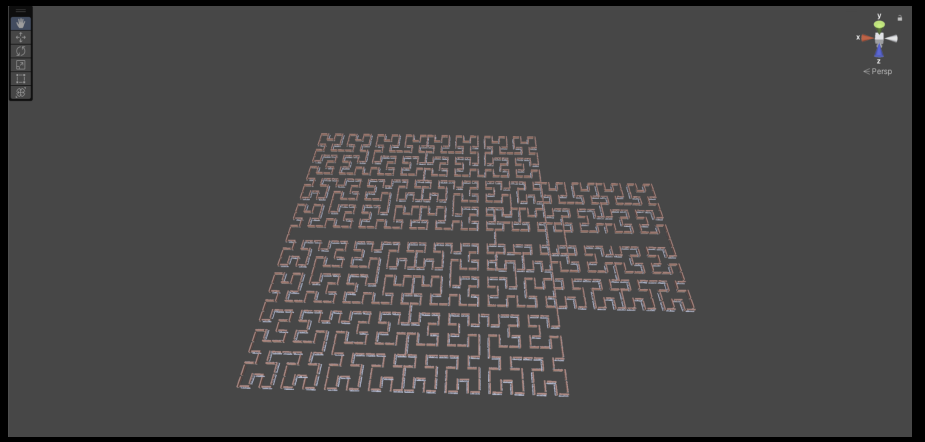


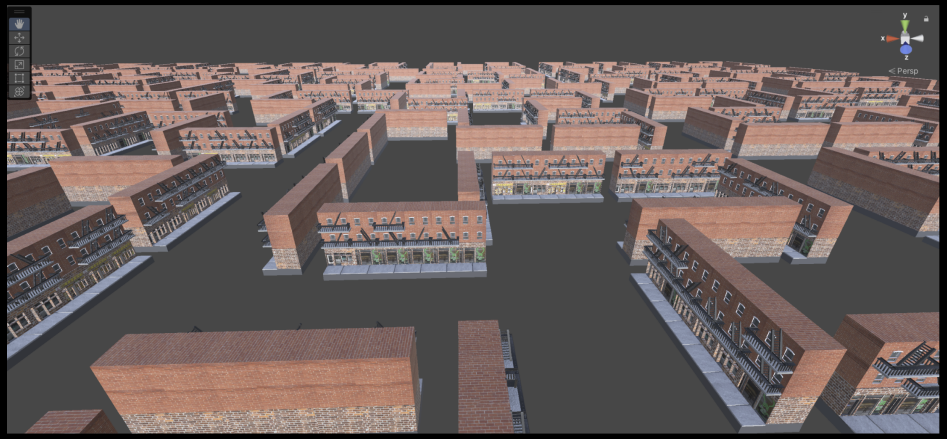
- Cell system rules for *Microsorium Development*

p_1 : $A_L \rightarrow A_R \uparrow (-45.0, 0.3)\, Q_1$
p_2 : $A_R \rightarrow Q_1 \uparrow (45.0, 0.7)\, A_L$
p_3 : $Q_1 \rightarrow Q_2 \uparrow (-98.0)\, Q_3$
p_4 : $Q_2 \rightarrow Q_1 \uparrow (90.0)\, Q_1$
p_5 : $Q_3 \rightarrow T \uparrow (0.0)\, T$
p_6 : $B_1 \rightarrow B_2 \uparrow (-90.0)\, T$
p_7 : $B_2 \rightarrow B_1 \uparrow (90.0)\, T$
p_8 : $C_0 \rightarrow C_1$
p_9 : $C_1 \rightarrow C_2 \uparrow (-90.0)\, T$
p_{10} : $C_2 \rightarrow T \uparrow (90.0)\, C_1$

- Implementation of Virtual Cities using L- Systems







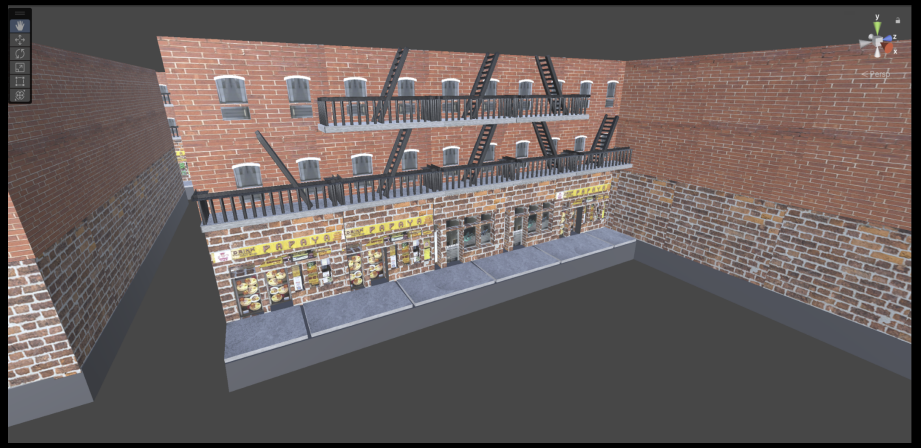


Image Courtesy: ESU Students - Briylnd and Kaitlin, SURE Grant2023

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Adam Runions



Elizabeth Barker



Jill Harrison
University of Bristol

HFSP,


THE END

- Thank you

Questions

• Post Production Survey

We greatly appreciate your participation and feedback to make the Provost Colloquium a better and more effective platform for the ESU community. Please provide your responses to the following short questions. It will take 1-3 minutes to take the survey. ***You do not need to sign in to Google***. Thank you very much!

